Should Competing Firms Reveal Their Capacity?

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Received 17 November 2011; revised 3 December 2012; accepted 4 December 2012
DOI 10.1002/nav.21521
Published online 9 January 2013 in Wiley Online Library (wileyonlinelibrary.com).

Abstract: In this article, we explore when firms have an incentive to hide (or reveal) their capacity information. We consider two firms that aim to maximize profits over time and face limited capacity. One or both of the firms have private information on their own capacity levels, and they update their beliefs about their rival's capacity based on their observation of the other firm's output. We focus on credible revelation mechanisms—a firm may signal its capacity through overproduction, compared to its myopic production levels. We characterize conditions when high-capacity firms may have the incentive and capability to signal their capacity levels by overproduction. We show that prior beliefs about capacity play a crucial, and surprisingly complex, role on whether the firm would prefer to reveal its capacity or not. A surprising result is that, despite the fact that it may be best for the high-capacity firm to overproduce to reveal its capacity when capacity information is private, it may end up with more profits than if all capacity information were public knowledge in the first place. © 2013 Wiley Periodicals, Inc. Naval Research Logistics 60: 64–86, 2013

Keywords: capacity management; volume competition; information asymmetry; game theory; Cournot game

1. INTRODUCTION

Capacity, which defines a firm's capability to meet market requirements, has long been identified as a key business metric. For example, when market outputs are the primary means of market competition (Cournot [8]), one player's supply level can be greatly influenced by its competitor's capacity: a firm acts nearly like a monopoly when its opponent has very limited capacity, whereas the firm behaves competitively when the rival's capacity is ample. Because any firm has to consider the impact of its own capacity on the competitor's business decisions, knowledge of competitors' capacity may be very valuable. Most literature in Economics and Operations Management (OM) assumes firms' capacities as common knowledge. This assumption, however, is not realistic in many scenarios—unlike financial information that a firm is required to reveal periodically to its shareholders and to the public—capacity information is private.

In practice, most firms attempt to actively influence knowledge about their capacity levels and capacity plans by either hiding the information (e.g., the authors of this article have signed many nondisclosure agreements, which specify that they will not reveal any information on firms' costs or capacities) or in some cases making announcements that appear to reveal the firm's capacity. Additionally, for many companies, there is a significant difference between the firm's "nominal" and "actual" capacity. The nominal capacity of a firm (or a plant) is what the firm or plant is expected to produce in a perfect environment (e.g., perfect quality, no yield loss, etc.). However, in many industries, yield loss or quality issues are chronic. For example, in the semiconductor industry, it is not unusual for a firm to achieve a yield of only 20 or 30 percent immediately after a plant is open. Even if the press or companies themselves make claims about a firm's capacity, it is not always obvious if these claims refer to nominal or actual capacity. As opposed to announcements, output of the firm may more credibly signal the firm's actual capacity.

We are interested in when firms have an incentive to signal that they have a "high" capacity (and when they may even be willing to incur a short-term loss to do so) and when they have an incentive to hide their capacity, and the mechanisms that firms can use to reveal or hide capacity information.

Our research addresses two interlinked questions: (1) Are firms better off with their competitors being uncertain about their actual capacity? and (2) How and when do firms want to signal their actual capacity to their competitors? We choose a stylized two-period Cournot game framework to explore these questions on capacity revelation. There are several markets that we believe are well-represented by our model.

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For example, gasoline prices are known to be heavily affected by regional refining capacity. Refinery shutdowns or refinery maintenance can significantly impact regional prices for gasoline (see, e.g., the recent report on the "Potential Impacts of Reductions in Refinery Activity on Northeast Petroleum Product Markets," by the Department of Energy [23]). Similarly, oil prices are influenced by oil producing countries' capacity to produce oil. However, it is very difficult for analysts to assess exactly how much capacity a country has at any moment (e.g., a recent Financial Times article (Blas [2]) quotes different parties speculating on Saudi Arabia's oil production capacity). In this kind of environment where capacity is not easily observable, but shipments of oil coming from Saudi Arabia are much easier to assess, actual quantity of oil produced is a credible signal of capacity. Dynamic Random-Access Memory (DRAM) memory prices are similarly significantly affected by available production capacity in the market and exhibit Cournot-like structures. Again, due to yield loss, equipment downtime, and so forth effective capacity of a memory plant is different than nominal capacity and output levels are credible signals of capacity.

It is well known in economics that in environments where capacity can be observed, firms may install excess capacity to deter entry by competitors (e.g., Dixit [10]). However, this result assumes that competitors have full knowledge of each other's available capacity. As indicated above, in many environments, the actual capacity of a competitor may be unobservable to a firm except through production levels of the competitor. Thus, our research contributes to this literature by exploring if firms may want to signal their capacity through overproduction to potentially decrease the intensity of future competition.

Although the analysis, examples, and intuition we provide are for a two-period model, we note that more generally, Period 2 in our model can be interpreted as a proxy for the discounted profits in the remainder of the product life cycle. For products produced over extended periods of time, an early signal may result in appropriately modified quantities for the remainder of the horizon. The 2-period setting allows us to capture the main tradeoff that we are interested in: When does a firm have an incentive to sacrifice short-term profits to credibly show that it really has capacity to influence future output decisions versus when does a firm have an incentive to hide capacity information? What are the mechanisms through which the firms may hide or reveal capacity? Would the firms be better off if all capacity information were public in the first place, instead of expanding resources to signal it?

In Section 2, we review the literature related to this problem. In Section 3, we describe our model for the case where capacity is one of two levels, high or low. We derive the necessary conditions for both pooling and separating equilibria. The separating equilibria correspond to situations where firms reveal their capacity information and pooling equilibria refer to the case where the capacity cannot be inferred from the output. We are particularly interested in situations where the firm would choose to overproduce to signal that it really has the capacity to influence its competitors' future production decisions. We first provide insights into these questions in Sections 3 and 4, through a simple model where only one firm's capacity is private. While the model in Sections 3 and 4 is the simplest model that captures the main issues we are interested in (the firm's capacity is binary, and there is only one-sided information asymmetry), in Section 5 we extend the basic model to 3 more complex cases with two-sided asymmetric information and with three possible levels of capacity (high, medium, and low). We show that many of the main insights survive these extensions while a few results differ. For example, in the case where capacity is binary, it is not economical for the high-capacity firm to overproduce to reveal its capacity if the price function is stationary or decreasing over time. However, when capacity types are more numerous, this is no longer true and a firm may overproduce to reveal capacity even under stationary or declining demand over periods. Another interesting result of our article is that even though firms may have to spend money to overproduce to reveal their capacity, they may actually end up making higher profits than if there was no information asymmetry in the first place. The surprising result here is that the asymmetry may benefit not only the lower-capacity firm but, in some cases, even a high-capacity firm, possibly shedding some light onto why firms take so much care on controlling the revelation or secrecy of their capacity. Finally, we consider an extension when a firm can first decide on how much capacity to invest in and, then, decides on how much to produce using this capacity. We show that our main insights hold as long as there is uncertainty about the firm's capacity investment costs. The article concludes in Section 6.

2. LITERATURE REVIEW

Capacity has an important role in making operation decisions. Many papers in operations have studied the effect of capacity constraints on inventory or production decisions. Some examples of this very large literature on the capacitated inventory management problem include Federgruen and Zipkin [11, 12] and Kapuscinski and Tayur [16]. There is also a rich literature on capacity management, that is, deciding on optimal capacity investment/disinvestment over time (Van Mieghem [24] provides an excellent review). However, most of the literature on production management with capacity constraints and on capacity investments assumes that the capacity of the firm is common knowledge.

There has been significant recent interest in OM recently on information asymmetry but most work to date focuses on demand or cost information asymmetry. For example, Ha [15] considers a model with one supplier and one manufacturer, where the manufacturer's cost structure is private information, and he shows that the supply chain cannot be coordinated under such information asymmetry. Cachon and Lariviere [3] analyze a case where manufacturer has better information about the demand forecast. The supplier decides on capacity based on the demand forecast provided by the manufacturer. They study how to design contracts that allow the supply chain to share demand forecasts credibly. Corbett [7] studies the effects of information asymmetries about supplier's setup costs and buyer's backorder costs respectively. Cachon and Netessine [4] give a summary of applications of information asymmetry in supply chain analysis. They consider mainly screening models, and they study the effects of demand or cost information asymmetry on designing truth-telling supply contracts.

More recently, Anand and Goyal [1] use a signaling game framework to study the issue of information sharing/leakage in a one-supplier-two-retailer setting. They show that under a wholesale price contract the supplier always leaks information to the entrant retailer. Kong et al. [17] consider a similar setting under a revenue sharing contract, and they demonstrate that a revenue sharing contract may prevent information leakage. Li et al. [20, 21] consider a supply chain with one supplier and one retailer, where the supplier also engages in a Cournot competition with the retailer through a direct channel. They study the effects of such supplier encroachment when the retailer has private demand information. They demonstrate that the retailer may purposely reduce or increase his order quantity to signal his demand information depending on the setting. In a different setting, Lai et al. [19] show that under a buy-back contract the buyer might overstock to reveal his demand prospects, and such stocking distortion hurts the buyer's profitability, and might or might not benefit the supplier and the supply chain. Similarly to OM literature, some previous economics literature deals with information asymmetry and signaling in an oligopoly/duopoly context, many of them with asymmetric information about demand or costs. Gal-Or [14] considers two duopolists who observe a signal of each others' unit-cost parameters before choosing their strategies. Riordan [22] studies a two-period Cournot model with demand uncertainty. The firms cannot observe the demand curve nor the previous quantity decisions of rivals. Based on past market prices, they infer the future demand. Two factors differentiate our article from the majority of the above literature. (1) We consider capacity information asymmetry, something that has not been analyzed in the literature and (2) we are interested in mechanisms of signaling capacity information and when and whether it is profitable to signal this information to one's competitor. Our model is a dynamic quantity game where two firms compete in the marketplace. The firms gain knowledge of their rival's capacity level by observing its output in the previous period. Essentially our model is a signaling game, and we study the impact of the capacity information asymmetry on the set of equilibria.

The two closest papers are Caminal [5] and Dana [9]. Caminal [5] considers a two-period duopoly game with price competition. In the first period, the players set their prices simultaneously and these are observed by both parties, then demands are realized. In the second period, based on observed prices, each firm updates its belief about the opponent's type (the type is the firm's "null demand"). The article discusses the impact of asymmetric information on the set of equilibria. Our work differs from Caminal [5] in the following: (1) We consider the information asymmetry in the firm's capacity; (2) We allow for nonstationary price function in the two periods; (3) We allow the firms to have different production margin costs; (4) Additionally, while Carminal imposes a restriction on the off-the-equilibrium-path beliefs, we relax this constraint by using the concept of Intuitive Criterion of Cho and Kreps [6] to refine the off-the-equilibrium-path beliefs.

Another related paper is Dana [9] with private information about inventories. The firms compete by setting prices, and deciding on inventory levels, consumers choose where to shop given firms' observable prices, and their expectations of firms' unobservable inventories. He shows in the Bertrand (price decision-first) model prices induce firms to hold more inventories, while in the Cournot (inventory decision-first) model the price can act as a signal of availability. He studies a symmetric setting, where all firms have identical costs, and considers symmetric equilibria of firms' price and availability decisions. Our model is different in that the firms have capacity constraints and the firms' quantity decision is limited by the firms' capacity. The output decision serves as a signal of the firms' capacity for the future, whereas Dana's model is a single-period two-stage game. Also, we allow firms to have different costs and the demand to change across periods. The fundamental difference, however, is that in our setting firms have to decide whether it is worthwhile to signal their capacity levels to their competitors by overproducing in the earlier period to gain advantage in a later period, which is not considered in Dana's model.

3. THE BASE MODEL

To gain insight into the role that capacity information asymmetry plays, we start with a base model, where there is information asymmetry about the capacity of one firm. (We will generalize our model later in multiple dimensions including having two-sided information asymmetry). In our base model, we consider two firms, 1 and 2, that produce one identical product. The firms compete across two periods—in each period, the firms simultaneously decide production quantities and sell at the market clearing price. Price functions for each period $t = 1, 2$ are:

$$p^t = a^t - b^t(Q_1^t + Q_2^t),$$

where Q_1^t and Q_2^t are production quantities of Firms 1 and 2 in period t, respectively. Both firms have limited capacities, denoted by K_1 and K_2. Firm 2's capacity, K_2, is known to Firm 1 (i.e., it is common knowledge). Firm 1, however, has private information about its own capacity K_1, and Firm 2 knows only the distribution of the capacity. To gain insight through a stylized model, we assume that two values are possible for firm 1's capacity $K_1 = \overline{K}$ or $\underline{K}$, with $\overline{K} > \underline{K}$. Throughout the article, Firm 1 with capacity $\overline{K}$ is referred to as a high-capacity firm or briefly high type, h, and $\underline{K}$ as a low-capacity firm or low type, l. (It is possible to generalize to more capacity types, e.g., low, medium, or high, but the main insights do not change as we shown in Section 5.2). Despite not knowing Firm 1's capacity, Firm 2 has certain initial beliefs on it. Specifically, at the beginning of the first period, Firm 2 believes $K_1 = \overline{K}$ with probability $\mu \in [0, 1]$, and $K_1 = \underline{K}$ with probability $1 - \mu$. All other information is common knowledge. The production costs are not necessarily the same and we use c_i to denote the production cost of firm i, for $i = 1, 2$. Thus, the total profit function of firm i is,

$$\Pi_i = [a^1 - b^1(Q_1^1 + Q_2^1) - c_i]Q_i^1 + [a^2 - b^2(Q_1^2 + Q_2^2) - c_i]Q_i^2.$$

The objective of each firm is to maximize the firm's profit across two periods.

Due to the uncertainty about Firm 1's capacity, Firm 2's production decision is dependent on its initial belief. Consequently, Firm 1's decision, depending on Firm 2's action, eventually is also a function of the initial beliefs of Firm 2. After observing the production quantities in Period 1, at the beginning of the second period, Firm 2 updates its belief about Firm 1's capacity. The two Cournot (quantity) games are not independent across periods. Both firms need to strategically decide their production quantities in Period 1, keeping in mind that they will influence beliefs of Firm 2 and, consequently, quantities and profits in Period 2.

To establish how beliefs about capacity influence the firms' production decisions and profits, we first analyze a one-period capacitated Cournot game with incomplete information. These results describe fully the second-period game and also establish general incentive patterns, useful in the first-period game. After solving the one-period game, we fully analyze the two-period model.

3.1. One-Period Model

For the analysis of one-period capacitated Cournot game with incomplete information, we omit the time subscript and denote the price function as $p = a - b(Q_1 + Q_2)$, where Q_1 and Q_2 are production quantities of Firms 1 and 2. As before, Firm 2 believes $K_1 = \overline{K}$ with probability μ, and $K_1 = \underline{K}$ with probability $1 - \mu$. Given that all decisions are a function of belief μ, let $Q_{1h}(\mu)$, $Q_{1l}(\mu)$, and $Q_2(\mu)$ denote the production quantities of Firm 1 with capacity $\overline{K}$, of Firm 1 with capacity $\underline{K}$ and of Firm 2, respectively. The profit functions of each firm are

$$\pi_{1\theta}(\mu) = [a - b(Q_{1\theta}(\mu) + Q_2(\mu)) - c_1]Q_{1\theta}(\mu), \quad \text{for } \theta = h, l,$$

$$\pi_2(\mu) = [a - b(\mu Q_{1h}(\mu) + (1 - \mu)Q_{1l}(\mu) + Q_2(\mu)) - c_2]Q_2(\mu),$$

where firm 2's profit reflects the uncertainty about Firm 1's capacity.

We use superscript $*$ to denote an equilibrium solution. To avoid trivial solutions, we assume that $0 < \mu < 1$, $a - 2c_1 + c_2 \geq 0$, and $a - 2c_2 + c_1 \geq 0$. Under these conditions, it is easy to show that first-order conditions combined with capacity bounds define equilibrium quantities, and that the equilibrium exists and is unique:

$$Q_{1h}^*(\mu) = \min\left\{\overline{K}, \frac{a - c_1}{2b} - \frac{1}{2}Q_2^*(\mu)\right\}, \tag{1}$$

$$Q_{1l}^*(\mu) = \min\left\{\underline{K}, \frac{a - c_1}{2b} - \frac{1}{2}Q_2^*(\mu)\right\}, \tag{2}$$

$$Q_2^*(\mu) = \min\left\{K_2, \frac{a - c_2}{2b} - \frac{\mu Q_{1h}^*(\mu) + (1 - \mu)Q_{1l}^*(\mu)}{2}\right\}. \tag{3}$$

Solving the above equations, we have the following two lemmas. Lemma 1 demonstrates that when Firm 1 is a high-capacity one, it benefits if its competitor knows this with certainty, while low capacity firm prefers ambiguity about its capacity.

LEMMA 1: If $K_2 > \frac{a-2c_2+c_1}{3b}$ and $\mu\overline{K} + (1 - \mu)\underline{K} > \frac{a-c_2}{b} - 2K_2$ and $\frac{(2-\mu)a-2c_2+\mu c_1}{(2-2\mu)b} - \frac{4-\mu}{2-2\mu}K_2 < \underline{K} \leq \frac{a-2c_1+c_2}{3b}$, then $\pi_{1h}^*(\mu)$ and $\pi_{1l}^*(\mu)$ are strictly increasing in μ. Otherwise, $\pi_{1h}^*(\mu)$ and $\pi_{1l}^*(\mu)$ are independent of μ.

PROOF: Please see the Appendix. □

In Fig. 1a the shaded region is defined by the conditions of Lemma 1, under which the equilibrium profits for both types are increasing in belief μ. In the remaining area, the equilibrium profits are independent of μ. Thus, either Firm 2's beliefs on Firm 1's capacity have no influence on Firm 1's profits, or both the low and high capacity Firm 1 benefit from being perceived as having high capacity. With the equilibrium payoffs derived in the proof of Lemma 1, we obtain the following property.

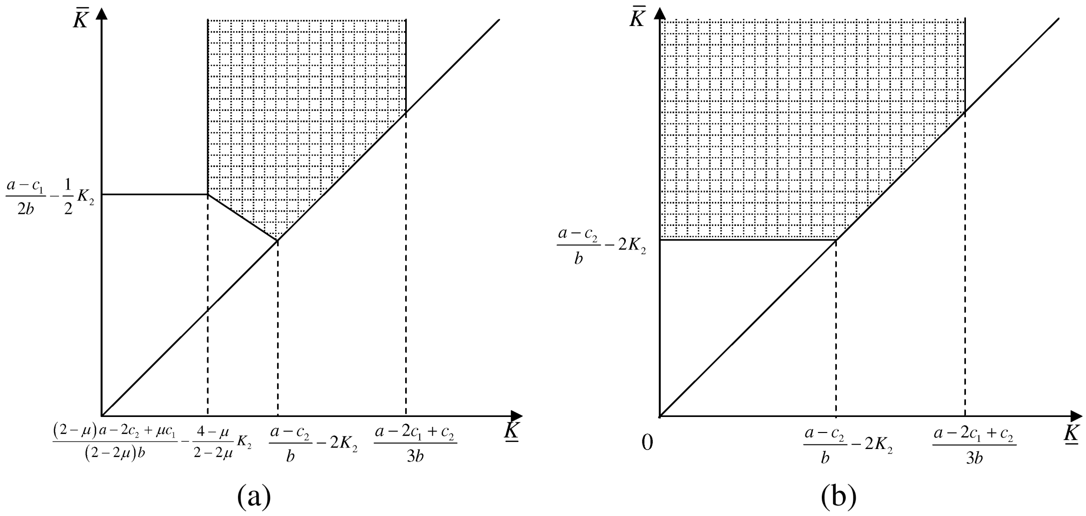

Figure 1. The effects of firm 2's belief μ on the equilibrium profits of both types, when $K_2 > \frac{a-2c_2+c_1}{3b}$ (a) Shaded area is where equilibrium profits for firm 1 increase in belief μ and (b) Shaded region illustrates necessary and sufficient conditions when high type gains more than low type from being perceived with probability 1 as high type.

LEMMA 2: Let $\mu \in (0,1)$. When $\frac{a-2c_2+c_1}{3b} < K_2$ and $\overline{K} > \frac{a-c_2}{b} - 2K_2$ and $\underline{K} < \frac{a-2c_1+c_2}{3b}$, for $\mu \in (0,1)$,

$$\pi^*_{1h}(1) - \pi^*_{1h}(\mu) > \pi^*_{1l}(1) - \pi^*_{1l}(\mu).$$

Otherwise,

$$\pi^*_{1h}(1) - \pi^*_{1h}(\mu) = \pi^*_{1l}(1) - \pi^*_{1l}(\mu).$$

PROOF: Follows from the proof of Lemma 1. □

Lemma 2 implies that the extra profits that the high-capacity Firm 1 gains if there were no uncertainty about its type, are always greater than or equal to the extra profits of the low-capacity Firm 1 if it were wrongly perceived to have high capacity. It also identifies the necessary and sufficient conditions for the inequality to be strict. These conditions are illustrated in Fig. 1b, as the shaded region. Lemma 2 plays a critical role in refining the off-the-equilibrium beliefs and, therefore, in our ability to fully characterize conditions when the high-capacity firm has an incentive to signal its capacity through overproduction in Period 1.

3.2. Two-Period Model

We are now ready to present the two-period game. In this subsection, we introduce the game and necessary conditions for pooling and separating equilibria to arise. We then use these conditions in Section 4 to derive a full characterization of the game.

We revert back to the two-period notation and use $Q^{t^*}_{1\theta}(\mu)$ and $Q^{t^*}_2(\mu)$ to denote the equilibrium quantities of Firm 1 of type $\theta = \{h,l\}$ and Firm 2 in period t, where Firm 2 believes that Firm 1 is high type with probability μ. $\pi^{t^*}_{1\theta}(\mu)$ and $\pi^{t^*}_2(\mu)$ are Firm 1's (of type θ) and Firm 2's profits under equilibrium, respectively.

Because, the beliefs are updated after Period 1, we use the concept of Perfect Bayesian Equilibrium and apply it to this two-period capacitated Cournot game with asymmetric information. Perfect Bayesian Equilibrium is a set of strategies and beliefs such that, at each stage of the game, strategies are optimal given the beliefs, and the beliefs are obtained from equilibrium strategies and observed actions using Bayes' rule (Fudenberg and Tirole [13]). The definition imposes no restrictions on the off-the-equilibrium path beliefs and, in many models, it results in multiple equilibria, some of them not reflecting any sound economic reality. Consequently, economic literature uses a range of refinements to impose minimal conditions that eliminate the abnormal behaviors (Fudenberg and Tirole [13]). In our model, we apply the Intuitive Criterion of Cho and Kreps [6] to refine the off-the-equilibrium beliefs. This is one of the most widely accepted criteria due to its intuitive appeal (as the label suggests).

We start with rephrasing the Intuitive Criterion in the context of our game. After observing that Firm 1 produces Q_1 in the first period, Firm 2's beliefs can be characterized by a probability measure $\Pr(T|Q_1)$, which denotes the probability that Firm 1's type is in set T, conditioned on observation Q_1. $T\setminus W$ denotes the set-theoretic difference between T and W. Define $\Pi_{1\theta}(Q_1, Q_2, \mu')$ to be the total profits for Firm 1 of type θ over two periods where Firms 1 and 2 produce Q_1 and Q_2 in Period 1, respectively, and the updated belief of Firm 2 after Period 1 is μ'.

DEFINITION 1 (Intuitive Criterion): Suppose the total equilibrium profits for Firm 1 of type θ over two periods are $\Pi^*_{1\theta}$, for $\theta \in \{h, l\}$. For any quantity Q_1, let $J(Q_1)$ be a subset of all types $\Theta = \{h, l\}$, such that

$$\Pi^*_{1\theta} > \max_{\mu':\Pr(\Theta|Q_1)=1} \Pi_{1\theta}(Q_1, Q_2^*, \mu').$$

If for this Q_1 there exists a $\theta' \in \Theta\backslash J(Q_1)$ such that

$$\Pi^*_{1\theta'} < \min_{\mu':\Pr(\Theta\backslash J(Q_1)|Q_1)=1} \Pi_{1\theta'}(Q_1, Q_2^*, \mu'),$$

then the equilibrium fails the Intuitive Criterion.

In the above definition, $J(Q_1)$ contains the types for which producing Q_1 cannot be optimal. Thus, $\Theta\backslash J(Q_1)$ are the types that might produce Q_1. The second condition, however, states that for at least one type $\theta' \in \Theta\backslash J(Q_1)$, producing Q_1, under any belief, results in profits strictly bigger than $\Pi^*_{1\theta}$. The equilibrium that jointly satisfies both conditions fails the intuitive criterion.

In this article, we focus only on pure strategy equilibria. Thus, it is possible to describe them in terms of pooling and separating equilibria. If the same quantity is produced by Firm 1, independently of its type (which is a pooling equilibrium), then Firm 2 cannot infer anything about K_1 and, thus, its belief about K_1 will not change. If, however, the quantities are different (which is a separating equilibrium), Firm 2 may deduce the Firm 1's type.

In the following, we first derive the necessary conditions for pooling and separating equilibria to arise, then apply those conditions to characterize the equilibria for the two-period model. We are particularly interested in whether Firm 1 has the incentive and capability to signal its capacity level.

3.2.1. Pooling Equilibria Conditions

In a pooling equilibrium, Firm 1 with low-or high-capacity produces the same quantity $Q_1^{1^*}$ in the first period, and no information can be inferred. Firm 2 produces $Q_2^{1^*}$. In the second period, Firm 1 of type $\theta, \theta = h, l$, produces $Q_{1\theta}^{2^*}(\mu'(Q_1^1))$, and Firm 2 produces $Q_2^{2^*}(\mu'(Q_1^1))$, where $\mu'(Q_1^1) \in [0, 1]$ is Firm 2's updated belief about Firm 1's type, after observing Q_1^1, the opponent's production in the first period. A pooling equilibrium is sustained by the following beliefs: If Q_1^1 is higher than low-type's capacity $\underline{K}$, Firm 1's type will be revealed, but if Q_1^1 is less than $\underline{K}$, no information is inferred[1]. That is,

$$\mu'(Q_1^1) = \begin{cases} 1 & \text{if } Q_1^1 > \underline{K} \\ \mu & \text{if } Q_1^1 \le \underline{K} \end{cases}.$$

[1] Note that the off-the-equilibrium belief can be relaxed to any value between 0 and 1 here. In the proofs of characterizing pooling equilibrium, the Intuitive Criterion is used to refine the off-the-equilibrium beliefs and no restriction is imposed on the off-the-equilibrium beliefs.

$\pi_1^1(Q_1^1, Q_2^1) = [a^1 - c_1 - b^1(Q_1^1 + Q_2^1)]Q_1^1$ is Firm 1's profit in the first period if each firm produces Q_1^1 and Q_2^1, respectively. A pooling equilibrium must satisfy the following conditions:

(PE1) Incentive compatibility constraint for low type: For any $Q_1^1 \le \underline{K}$,

$$\pi_1^1(Q_1^1, Q_2^{1^*}) - \pi_1^1(Q_1^{1^*}, Q_2^{1^*}) \le \pi_{1l}^{2^*}(\mu) - \pi_{1l}^{2^*}(\mu'(Q_1^1)),$$

that is, if Firm 1 is low type, its profit increase in Period 1 resulting from deviating to quantity Q_1^1 from $Q_1^{1^*}$ is smaller than the loss in Period 2 due to the change in Firm 2' belief.

(PE2) Incentive compatibility constraint for high type: For any $Q_1^1 \le \overline{K}$,

$$\pi_1^1(Q_1^{1^*}, Q_2^{1^*}) - \pi_1^1(Q_1^1, Q_2^{1^*}) \ge \pi_{1h}^{2^*}(\mu'(Q_1^1)) - \pi_{1h}^{2^*}(\mu),$$

that is, if Firm 1 is high type, its profit loss in Period 1 resulting from deviating to quantity Q_1^1 from $Q_1^{1^*}$ is greater than the additional profits in Period 2 resulting from changing Firm 2's belief.

(PE3) Individual rationality constraint for firm 2:[2]

$$Q_2^{1^*} = \min\left\{K_2, \frac{a^1 - c_2}{2b^1} - \frac{1}{2}Q_1^{1^*}\right\}.$$

It implies that in Period 1, Firm 2 produces the quantity that maximizes its one-period profits.

(PE4) Capacity constraint:

$$Q_1^{1^*} \le \underline{K} \text{ and } Q_2^{1^*} \le K_2.$$

In our article, we apply the Intuitive Criterion of Cho and Kreps [6] to refine the off-the-equilibrium beliefs. We will call the pooling equilibria that survive the Intuitive Criterion, **intuitive pooling equilibria**.

Directly from Definition 1, a pooling equilibrium fails the Intuitive Criterion if there exists a quantity Q_1^1 such that

$$\Pi^*_{1l}(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu) > \Pi_{1l}(Q_1^1, Q_2^{1^*}, \mu' = 1)$$

and

$$\Pi^*_{1h}(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu) < \Pi_{1h}(Q_1^1, Q_2^{1^*}, \mu' = 1).$$

The rationale for the intuitive criterion is as follows. Consider a pooling equilibrium where each type produces $Q_1^{1^*}$, Firm 2 produces $Q_2^{1^*}$, and no information is inferred. If the above two conditions are satisfied, Firm 2 can infer that a deviation to Q_1^1 indicates a high type, because by the first inequality the low type gets lower profits than the equilibrium profits, no matter how Firm 2 updates the belief. The high type, however, could be better off by producing Q_1^1 due to the second inequality. Therefore, the pooling equilibrium will not be sustained.

[2] It is not necessary to impose individual rationality for high-type Firm 1. Instead, this condition is derived.

3.2.2. Separating Equilibria Conditions

In a separating equilibrium, depending on its type (low or high), Firm 1 produces different quantities in the first period. This allows Firm 2 to deduce its rival's type. Any separating equilibria will take the following form: In the first period, Firm 1 of type θ, $\theta = h, l$, produces $Q_{1\theta}^{1^*}$, and Firm 2 produces $Q_2^{1^*}$. In the second period, Firm 1 of type θ produces $Q_{1\theta}^{2^*}(\mu'(Q_1^1))$, and Firm 2 produces $Q_2^{2^*}(\mu'(Q_1^1))$, where $\mu'(Q_1^1) \in \{0, 1\}$ is the updated belief of Firm 2 about Firm 1's type after observing, Q_1^1, the opponent's production in the first period. A separating equilibrium is sustained by the following beliefs: If the quantity Q_1^1 is above $\underline{K}$ Firm 2 knows Firm 1's type in the second period, if Q_1^1 is greater or equal to $Q_{1h}^{1^*}$ then Firm 2 will interpret it as coming from a high-type firm, and any quantity below $Q_{1h}^{1^*}$ as coming from a low-type firm.[3] That is,

$$\mu'(Q_1^1) = \begin{cases} 1 & \text{if } Q_1^1 > \underline{K} \\ 1 & \text{if } Q_1^1 \geq Q_{1h}^{1^*} \\ 0 & \text{if } Q_1^1 < Q_{1h}^{1^*} \end{cases}.$$

In general, a separating equilibrium must satisfy the following necessary conditions:

(SE1) Incentive compatibility constraint for low type: For any $Q_1^1 \leq \underline{K}$,

$$\pi_1^1(Q_{1l}^{1^*}, Q_2^{1^*}) - \pi_1^1(Q_1^1, Q_2^{1^*}) \geq \pi_{1l}^{2^*}(\mu'(Q_1^1)) - \pi_{1l}^{2^*}(0),$$

that is, if Firm 1 is low type, its loss of profit in Period 1 resulting from producing Q_1^1 instead of $Q_{1l}^{1^*}$ is greater than any increase in profit in Period 2 due to changing Firm 2's belief.

(SE2) Incentive compatibility constraint for high type: For any $Q_1^1 \leq \overline{K}$,

$$\pi_1^1(Q_1^1, Q_2^{1^*}) - \pi_1^1(Q_{1h}^{1^*}, Q_2^{1^*}) \leq \pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(\mu'(Q_1^1)),$$

that is, for the high type, any increase in Period-1 profit from producing Q_1^1 instead of $Q_{1h}^{1^*}$ is less than the decrease in profit resulting from changing Firm 2's belief in Period 2.

(SE3) Individual rationality constraint for Firm 1 of low type:[4]

$$Q_{1l}^{1^*} = \min\left\{\underline{K}, \frac{a^1 - c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}\right\},$$

that is, if the resulting equilibrium is a separating one, then in the first period, low-type Firm 1 must have produced quantity that maximizes one-period profits.

(SE4) Individual rationality constraint for Firm 2:

$$Q_2^{1^*} = \min\left\{K_2, \frac{a^1 - c_2}{2b^1} - \frac{\mu Q_{1h}^{1^*} + (1-\mu)Q_{1l}^{1^*}}{2}\right\}.$$

Similarly, in Period 1, Firm 2 will produce the quantity that maximizes its one-period expected profits.

(SE5) Capacity constraint:

$$Q_{1h}^{1^*} \leq \overline{K},\ Q_{1l}^{1^*} \leq \underline{K}, \quad \text{and} \quad Q_2^{1^*} \leq K_2.$$

In the next section, we apply the above necessary conditions for pooling and separating equilibria to fully characterize the equilibria.

4. PRIMARY RESULTS

We start our analysis by noting that the two periods are not necessarily identical (as reflected through the coefficients of price functions, which could differ significantly across periods). It is convenient to use uncapacitated one-period Cournot outcomes to compare market sizes across periods. Note that $\frac{a^1-2c_1+c_2}{3b^1}$ and $\frac{a^2-2c_1+c_2}{3b^2}$ are the equilibrium quantities for Firm 1 in a game with infinite capacity. We have two possibilities:

1. larger market potential in the first period: $\frac{a^1-2c_1+c_2}{3b^1} \geq \frac{a^2-2c_1+c_2}{3b^2}$, or
2. larger market potential in the second period: $\frac{a^1-2c_1+c_2}{3b^1} < \frac{a^2-2c_1+c_2}{3b^2}$.

Our analysis will show that these cases are qualitatively different. In case (1), each player produces its myopic optimal quantity in Period 1. That is, the firm does not take into account the fact that the game will be played in Period 2 and only optimizes its decision for the current period based on current beliefs. In case (2), however, under certain conditions, the high type will benefit when deviating from its myopically optimal quantity in the first period. This is done by producing more than its myopic quantity to differentiate itself from the low type.

4.1. Smaller Market Potential in the Second Period

The following theorem demonstrates that, when the market potential in Period 1 is at least as large as that in Period 2, each firm will choose its production quantities myopically,

[3] Here, we provide one complete belief specification in characterizing the separating equilibrium. There exist many other off-equilibrium specifications which result in a separating equilibrium. Furthermore, note that our assumed belief specification implies that the high-type Firm 1 chooses a higher quantity than the low-type Firm 1 at a separating equilibrium, which we will verify in the following proofs when we fully characterize the separating equilibrium.

[4] High-type Firm 1 has no rationality constraint in separating equilibrium, as it is interested in differentiating itself from the low type. Therefore, high-type firm will not necessarily follow the production imposed by individual rationality.

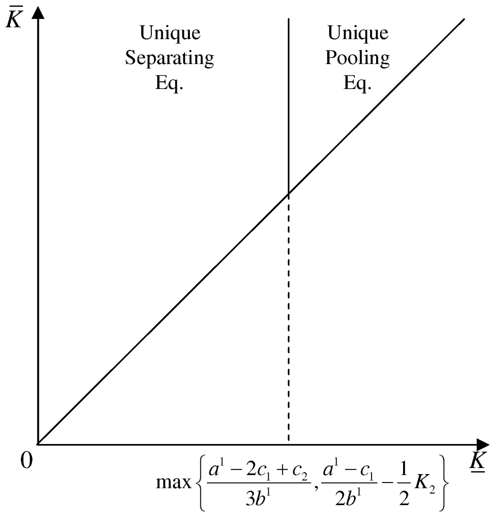

Figure 2. Equilibrium partition when $\frac{a^1-2c_1+c_2}{3b^1} \geq \frac{a^2-2c_1+c_2}{3b^2}$.

that is, ignore the effect of its quantity decision in Period 1 on the belief updates.

THEOREM 1: When $\frac{a^1-2c_1+c_2}{3b^1} \geq \frac{a^2-2c_1+c_2}{3b^2}$, the equilibrium solution for Period 1 is unique and myopic.

a. If $\underline{K} < \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, the equilibrium is a separating one. Firm 1 of low type produces $Q_{1l}^{1^*}(\mu)$, Firm 1 of high type produces $Q_{1h}^{1^*}(\mu)$ ($\neq Q_{1l}^{1^*}(\mu)$), and Firm 2 produces $Q_2^{1^*}(\mu)$.
b. If $\underline{K} \geq \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, the equilibrium is an intuitive pooling one. Each type produces $Q_1^{1^*} = Q_{1h}^{1^*}(\mu) = Q_{1l}^{1^*}(\mu)$, and Firm 2 produces $Q_2^{1^*} = Q_2^{1^*}(\mu)$.

PROOF: Please see the Appendix. □

Figure 2 illustrates the nature of equilibria obtained in Period 1 when market potential is smaller in Period 2: If the low type's capacity is smaller than the threshold derived in Theorem 1, the two types produce different quantities in Period 1 and, therefore, their types are revealed. Thus, the separation in that case is driven by the lack of sufficient capacity of the low type rather than any special strategic behavior of the high type. If, however, the low type's capacity is large enough, both types produce the same quantity in Period 1, and no information is revealed. Note also that the threshold dividing separating and pooling equilibria depends explicitly on the capacity of firm 2.

If K_2 is small ($K_2 \leq \frac{a^2-2c_2+c_1}{3b^2}$), in the second period Firm 2's belief about Firm 1's type will affect neither the high-type nor low-type firm's profit under equilibrium. Therefore, in Period 1, each firm produces its one-period myopic best response quantity. However, if K_2 is large ($K_2 > \frac{a^2-2c_2+c_1}{3b^2}$), by Lemma 1, Firm 2's belief may influence the second-period actions, and needs to be explicitly taken into account in Period 1. Despite this potential influence, when capacity of the low type, $\underline{K}$, is small, because the high-type's myopic optimal level is higher than the low-type's capacity, its type will be revealed. Thus, the low type also produces its one-period myopic quantity because it has no chance to imitate the high type. On the other hand, when $\underline{K}$ is large, Firm 2's belief about Firm 1's type does not affect each player's equilibrium profit in the second period, therefore, each type will produce its myopic best response again in the first period.

The above intuition has significant consequences. If we face declining market potential over time, either the "beliefs" on capacity are not relevant, or they are relevant but not "actionable," because it is infeasible for the low-capacity manufacturer to imitate the high-type manufacturer. Thus, under these conditions either:

1. the optimal policies separate the high and low types and the two types produce myopic optimal quantities, or
2. the optimal policy is pooling and has no influence on Period 2 profits (due to the fact that capacity is large enough and the incentive to differentiate does not exist).

In this case, the firm needs to take no special action to hide or reveal its capacity: deciding quantity myopically is optimal. As we will see, this is not true if the second period has larger market potential.

4.2. Larger Market Potential in the Second Period

Recall that larger market potential in the second period means that the Cournot output without capacity limitation in the second period is higher than that in the first period. Due to potentially higher profits in the second period, the high type might have a stronger incentive to differentiate itself in the first period, and possibly sacrifice some short-term profits to signal its type. The next theorems describe when this can or cannot happen. First, Theorem 2 shows that, if Firm 2 has low enough capacity, K_2, then once again the beliefs on capacity do not matter. In this case, Firm 1 has no incentive to signal that it has high capacity because Firm 2's capacity is low enough that Firm 2's belief will not be able to affect Firm 1's optimal behavior in the second period.

THEOREM 2: When $\frac{a^1-2c_1+c_2}{3b^1} < \frac{a^2-2c_1+c_2}{3b^2}$ and $K_2 \leq \frac{a^2-2c_2+c_1}{3b^2}$,

a. If $\underline{K} < \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, the equilibrium is a separating one. Firm 1 of low type produces $Q_{1l}^{1^*}(\mu)$, Firm 1 of high type produces $Q_{1h}^{1^*}(\mu)$ ($\neq Q_{1l}^{1^*}(\mu)$), and Firm 2 produces $Q_2^{1^*} = Q_2^{1^*}(\mu)$.

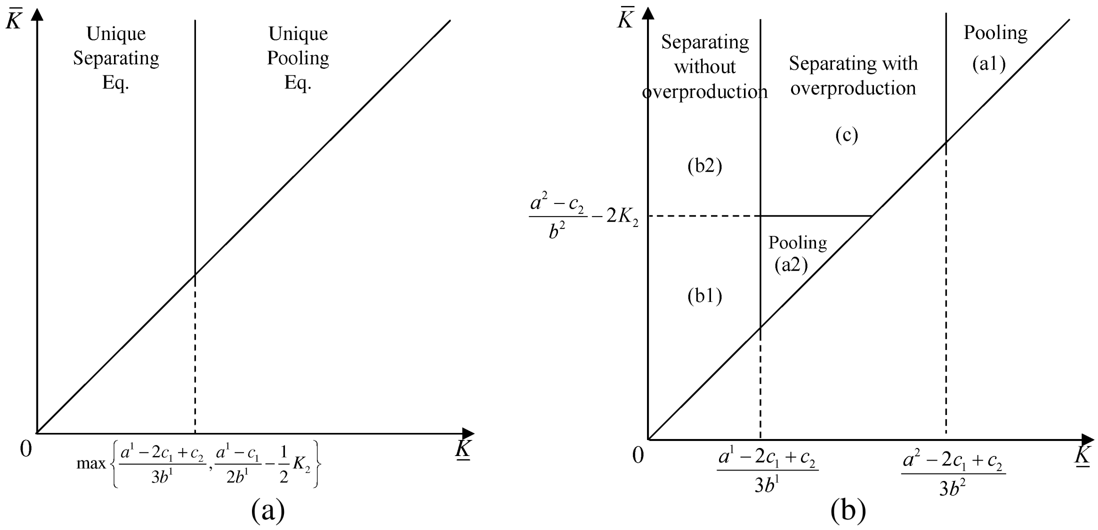

Figure 3. Equilibrium partition when $\frac{a^1-2c_1+c_2}{3b^1} < \frac{a^2-2c_1+c_2}{3b^2}$ (a) $K_2 \le \frac{a^2-2c_2+c_1}{3b^2}$ and (b) $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$.

b. If $\underline{K} \ge \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, the equilibrium is an intuitive pooling equilibrium. Each type produces $Q_1^{1^*} = Q_{1h}^{1^*}(\mu) = Q_{1l}^{1^*}(\mu)$, and Firm 2 produces $Q_2^{1^*} = Q_2^{1^*}(\mu)$.

PROOF: The proof is similar to the proof of Theorem 1 and, therefore, omitted. □

As stated in Theorem 2, and shown in Fig. 3a, if K_2 is small, in the second period, Firm 2's belief about Firm 1's type will not affect each player's profit under equilibrium because both types of Firm 1 know that Firm 2 will use its full capacity K_2, and thus both types produce their best response to Firm 2's output K_2 regardless of Firm 2's belief about their types. In this case, each player acts myopically in the first period and produces its one-period best response quantity. On the other hand, the next theorem characterizes how Firm 1 adjusts its production to signal its capacity when Firm 2 has sufficient capacity in the second period.

THEOREM 3: When $\frac{a^1-2c_1+c_2}{3b^1} < \frac{a^2-2c_1+c_2}{3b^2}$ and $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$,

a. If $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$, the equilibrium is a separating one. Firm 1 of low type produces $Q_{1l}^{1^*}(\mu)$, Firm 1 of high type produces $Q_{1h}^{1^*}(\mu)$ ($\neq Q_{1l}^{1^*}(\mu)$), and Firm 2 produces $Q_2^{1^*}(\mu)$.
b. If $\underline{K} \ge \frac{a^2-2c_1+c_2}{3b^2}$ or $\{\overline{K} \le \frac{a^2-c_2}{b^2} - 2K_2$ and $\underline{K} \ge \frac{a^1-2c_1+c_2}{3b^1})\}$, the equilibrium is an intuitive pooling equilibrium. Each type produces $Q_1^{1^*} = Q_{1h}^{1^*}(\mu) = Q_{1l}^{1^*}(\mu)$ for $\theta = h, l$, and Firm 2 produces $Q_2^{1^*} = Q_2^{1^*}(\mu)$.
c. If $\frac{a^1-2c_1+c_2}{3b^1} \le \underline{K} < \frac{a^2-2c_1+c_2}{3b^2}$ and $\overline{K} > \frac{a^2-c_2}{b^2} - 2K_2$, the equilibrium is a separating one, with

$$Q_{1h}^{1^*} = \min\left\{\underline{K}^+, \frac{a^1-2c_1+c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0)}{b^1}}\right\},$$

$$Q_{1l}^{1^*} = \frac{a^1-2c_1+c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu} \times \min\left\{\underline{K}^+, \frac{a^1-2c_1+c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0)}{b^1}}\right\},$$

$$Q_2^{1^*} = \frac{(1+\mu)a^1 - 2c_2 + (1-\mu)c_1}{(3+\mu)b^1} - \frac{2\mu}{3+\mu}\min\left\{\underline{K}^+, \frac{a^1-2c_1+c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0)}{b^1}}\right\}.$$

where $\underline{K}^+$ is infinitesimally above $\underline{K}$.

PROOF: Please see the Appendix. □

The production decisions become interesting and complex when K_2 is large enough (i.e., $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$) as shown in Fig. 3b. In this case, Firm 2 may choose a quantity below its capacity K_2 depending on his belief of Firm 1's type. Thus, Firm 1 may have an incentive to shape that belief through its first-period production decision. We provide an explanation of the resulting equilibria below.

- In area (a1), $\underline{K}$ (the low type's capacity) is high enough so that Firm 2 would take the same action regardless of which type it faced in the second period. Similarly, in areas (a2) and (b1), the high type's capacity, $\overline{K}$ is low enough that Firm 2 would use its full capacity in the second period regardless of what type firm it faced. Thus, in these regions, Firm 1 behaves myopically. In region (a2), myopic behavior results in the two types producing the same quantity, and thus we have a pooling equilibrium. However, in region (b1), Firm 1 of low type does not have enough capacity to produce the myopic quantity and thus produces an amount equal to its capacity, $\underline{K}$. Thus, the equilibrium separates the high and the low type even though the two types just act myopically. Similar separation takes place in region (b2). Firm 1 of high type has high enough capacity and would like to differentiate itself from the low type. However, if the high type just produces its one-period myopic quantity, the low type cannot match it and, thus, the high type can differentiate itself through just following its myopic policy.
- Area (c) is the most interesting. In this case, like (b2), Firm 1 of the high type has high enough capacity to signal its type to Firm 2. However, if it produces its myopically optimal quantity in Period 1, the low type **can** match it. The high type is able to and decides to produce at a level higher than its myopic quantity. The level is chosen so that it would not be economical for the low type to produce at this level. Thus, the high type signals to Firm 2 its type by producing above its myopic quantity so that Firm 2 can deduce that it must be the high type because the low type could not profit by deviating to this level.

Capacity determines the upper limit of a firm's production output, and the impact of capacity information on the other operational decisions is crucial only when the projected capacity is low compared to the market potential. From Theorem 1(b), 2(b), and 3(b), the pooling equilibria exist and survive the Intuitive Criterion only when the lowest possible capacity level, $\underline{K}$, is high enough so that Firm 2's belief about Firm 1's type in the second period does not influence Firm 1's equilibrium payoff. Basically, Firm 1's capacity is high enough under both scenarios that Firm 1 would produce the same quantity in each case and therefore what Firm 1 produces preserves the asymmetry of information about Firm 1's capacity. Therefore, each firm will play its one-period best response quantity in the first stage which results in a pooling equilibrium. In all other cases, the separating equilibrium is the only equilibrium which survives the Intuitive Criterion refinement. This is consistent with the standard use of Intuitive Criterion which eliminates pooling equilibria. Our characterization of the equilibrium solution suggests that the high type desires to distinguish itself from the low type, and in certain cases, would even forgo short-term profits to do so. In the base model, this only happens when the market potential for the second period is larger than the first period. However, in Section 5.2, we will show that, when we generalize our model to more types than two (e.g., a high-, medium-, or low-capacity firm), the high-type firm may want to distinguish itself by overproducing even when the market potential is not higher in Period 2. Thus, this kind of interesting behavior can occur in many situations. It would also be interesting to analyze who benefits from the uncertainty about the capacity level. Initial intuition may suggest that the low-type firm always benefits, whereas the high-type would always prefer a situation where its type was known in the first place (after all, we just showed that it may be optimal for the high-type firm to overproduce to ensure that its competitor is sure about its capacity). In the next subsection, we explore this question by comparing the results in this section to the case where all firms' capacity levels are known in advance. Surprisingly, we show that even though the high-type firm may have to overproduce to ensure its capacity is known by its competitor, it may still have higher profits compared to the case where its capacity was known by its competitor in advance.

4.3. Who Benefits from Uncertainty over Capacity?

In this section, we compare the equilibrium quantity decisions and profits for two situations: one is the two-period model, considered in the previous section, and the other is a two-period model with full information, where Firm 1's type is known at the beginning of Period 1. Clearly, with full information, the two firms play two Cournot games independently in each period, thus choosing their myopic quantity levels in each period. Our aim is to explore which firm benefits from ambiguity about capacity.

Table 1 lists the equilibrium quantities for each firm in the first period for both models. Note that, in the region where the high type differentiates itself from the low type, the high type produces more than its myopic optimal level. Consequently, in this region, Firm 2, not knowing yet the type of the competitor, produces less than its myopic optimal level anticipating a possibility of "overproduction" by the high type. This further implies that the low type is able to take advantage of Firm 2's concession, and thus produces more than its myopic level, as well.

Table 2 shows the total profits over two periods for all firms. Information asymmetry always benefits the low type, as one

Table 1. Comparison of the equilibrium quantity decisions in Period 1.

		Asymmetric information		Full information
Separating	$Q_{1h}^{1^*}$	$Q_{1h}^{1^*}(\mu)$	$\leq$	$Q_{1h}^{1^*}(1)$
(w/o overproduction)	$Q_{1l}^{1^*}$	$Q_{1l}^{1^*}(\mu)$	$\geq$	$Q_{1l}^{1^*}(0)$
	$Q_2^{1^*}$	$Q_2^{1^*}(\mu)$	$\geq$	$Q_2^{1^*}(1)$ if firm 1 is high type
			$\leq$	$Q_2^{1^*}(0)$ if firm 1 is low type
Separating	$Q_{1h}^{1^*}$	$Q_{1h}^{1^*}(\mu) + \frac{3+\mu}{3}\sqrt{\frac{\Delta}{b^1}}$	$\geq$	$Q_{1h}^{1^*}(1)$
(w/ overproduction)	$Q_{1l}^{1^*}$	$Q_{1l}^{1^*}(\mu) + \frac{\mu}{3}\sqrt{\frac{\Delta}{b^1}}$	$\geq$	$Q_{1l}^{1^*}(0)$
	$Q_2^{1^*}$	$Q_2^{1^*}(\mu) - \frac{2\mu}{3}\sqrt{\frac{\Delta}{b^1}}$	$\leq$	$Q_2^{1^*}(1) = Q_2^{1^*}(0)$
Pooling	$Q_{1h}^{1^*}$	$Q_{1h}^{1^*}(\mu)$	$=$	$Q_{1h}^{1^*}(1)$
	$Q_{1l}^{1^*}$	$Q_{1l}^{1^*}(\mu)$	$=$	$Q_{1h}^{1^*}(0)$
	$Q_2^{1^*}$	$Q_2^{1^*}(\mu)$	$=$	$Q_2^{1^*}(1) = Q_2^{1^*}(0)$

$Q_{1\theta}^{1^*}$ = type θ's myopic equilibrium quantity in Period 1 with belief μ for $\theta = h, l$.
$Q_2^{1^*}$ = Firm 2's myopic equilibrium quantity in Period 1 with belief μ.
$\Delta = \pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0)$ = extra profits low type obtains in Period 2 by being perceived as a high type.

may expect. Surprisingly, the high-type firms may actually also strictly benefit from information asymmetry compared to the case where the firm's capacity was known in the first place. This is shown in the first line of the case where the high type differentiates itself in Table 2. In this case, Firm 1 with high capacity overproduces to show that it is high type. The expectation that Firm 1 will overproduce may actually lead Firm 2 to underproduce in the first period and, therefore, Firm 1 can in some cases recoup the costs it incurred overproducing in the first period. Thus, Firm 1 can end up

Table 2. Comparison of the total profits under equilibrium.

		Asymmetric information		Full information
Separating	Π_{1h}^*	$\pi_{1h}^{1^*}(\mu) + \pi_{1h}^{2^*}(1)$	$\leq$	$\pi_{1h}^{1^*}(1) + \pi_{1h}^{2^*}(1)$
(w/o overproduction)	Π_{1l}^*	$\pi_{1l}^{1^*}(\mu) + \pi_{1l}^{2^*}(0)$	$\geq$	$\pi_{1l}^{1^*}(0) + \pi_{1l}^{2^*}(0)$
	Π_2^*	$\pi_2^{1^*}(\mu) + \mu\pi_2^{2^*}(1) + (1-\mu)\pi_2^{2^*}(0)$	$\leq$ or $\geq$	$\mu\pi_2^{1^*}(1) + (1-\mu)\pi_2^{1^*}(0)$ $+\mu\pi_2^{2^*}(1) + (1-\mu)\pi_2^{2^*}(0)$
	Π_{1h}^*	$b^1\left(Q_{1h}^{1^*}(\mu) + \frac{\mu}{3}\sqrt{\frac{\Delta}{b^1}}\right)^2$ $-\Delta + \pi_{1h}^{2^*}(1)$	$\leq$ or $\geq$	$\pi_{1h}^{1^*}(1) + \pi_{1h}^{2^*}(1)$
Separating (w/ overproduction)	Π_{1l}^*	$b^1\left(Q_{1l}^{1^*}(\mu) + \frac{\mu}{3}\sqrt{\frac{\Delta}{b^1}}\right)^2$ $+\pi_{1l}^{2^*}(0)$	$\geq$	$\pi_{1l}^{1^*}(0) + \pi_{1l}^{2^*}(0)$
	Π_2^*	$b^1\left(Q_2^{1^*}(\mu) - \frac{2\mu}{3}\sqrt{\frac{\Delta}{b^1}}\right)^2$ $+\mu\pi_2^{2^*}(1) + (1-\mu)\pi_2^{2^*}(0)$	$\leq$	$\mu\pi_2^{1^*}(1) + (1-\mu)\pi_2^{1^*}(0)$ $+\mu\pi_2^{2^*}(1) + (1-\mu)\pi_2^{2^*}(0)$
Pooling	Π_{1h}^*	$\pi_{1h}^{1^*}(\mu) + \pi_{1h}^{2^*}(\mu)$	$=$	$\pi_{1h}^{1^*}(1) + \pi_{1h}^{2^*}(1)$
	Π_{1l}^*	$\pi_{1l}^{1^*}(\mu) + \pi_{1l}^{2^*}(\mu)$	$=$	$\pi_{1l}^{1^*}(0) + \pi_{1l}^{2^*}(0)$
	Π_2^*	$\pi_2^{1^*}(\mu) + \pi_2^{2^*}(\mu)$	$=$	$\mu\pi_2^{1^*}(1) + (1-\mu)\pi_2^{1^*}(0)$ $+\mu\pi_2^{2^*}(1) + (1-\mu)\pi_2^{2^*}(0)$

For $\theta = h, l$ and $t = 1, 2$.
$\pi_{1\theta}^{t^*}$ = type θ's myopic equilibrium profits in period t with belief μ and price parameter (a^t, b^t).
$\pi_2^{t^*}$ = firm 2's myopic equilibrium profits in period t with belief μ and price parameter (a^t, b^t).
$\Delta = \pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0)$ = extra profits that low type gets in Period 2 by being perceived as high type.

Table 3. Example that the high-type Firm 1 benefits from information asymmetry ($a^1 = 30, a^2 = 90, b^1 = b^2 = 1, c_1 = c_2 = 0, \overline{K} = 35, \underline{K} = 29.8, K_2 = 35, \mu = 0.5$).

	Asymmetric information					Full information				
	Q_1^1	Q_1^2	π^1	π^2	Π	Q_1^1	Q_1^2	π^1	π^2	Π
Firm 1 ($\theta = h$)	12.014	30	102.85	900	1002.85	10	30	100	900	1000
Firm 1 ($\theta = l$)	10.288	29.8	105.83	896.98	1002.81	10	29.8	100	896.98	996.98
Firm 2	9.425	30 (h) 30.1 (l)	88.82	903.01	991.83	10	30 (h) 30.1 (l)	100	903.01	1003.01

being better off having to spend money to signal it is high type compared to the case when the fact that it is high type was already known.

Consider the numerical example in Table 3 where the price functions in two periods are $p^1 = 30 - (Q_1^1 + Q_2^1)$ and $p^2 = 90 - (Q_1^2 + Q_2^2)$, $c_1 = c_2 = 0$, $\overline{K} = 35$, $\underline{K} = 29.8$, $K_2 = 35$, and Firm 2's initial belief on Firm 1's capacity is $\mu = 0.5$. In this example, the high type overproduces to quantity 12.014 (instead of myopic Solution 10) to signal its type in the first period. The expected total profits over two periods for the high-type Firm 1 are 1002.85 when Firm 2 has uncertainty over Firm 1's capacity and 1000 where all capacity is public knowledge from the beginning. Thus, in this case, Firm 1 benefits from capacity ambiguity despite having to overproduce to reveal its type.

Thus, we conclude that firms may have good reasons to hide their capacity information. The firm with the low capacity always benefits from ambiguity, but as we saw above, the firm with high capacity may also be better off with capacity ambiguity even if it has to spend resources to signal its capacity. Interestingly, even Firm 2, whose capacity is known in this case, may sometimes benefit from ambiguity of the opposing firm's capacity. For another illustration of less intuitive effects of information asymmetry, consider the example in Table 4. Comparing the case when Firm 2 knows Firm 1's capacity to the case where it does not, we see that Firm 2 is able to get higher profits in the asymmetric information case. Notice that Firm 1 of the high type produces at its myopic optimal under the full-information case, but has to decrease that level when it faces a Firm 2 who is unsure about Firm 1's capacity level. Firm 2 takes advantage of that and is able to increase its profits.

The interesting result here is that firms may benefit from capacity ambiguity even if they have to spend money to then reveal their capacity compared to the case when their capacity was known in the first place. In this section, we limited ourselves to the capacity of the firm being high or low (e.g., two types), and observed that the high-type firm would only overproduce to signal its capacity when it is facing a higher second-period demand. Interestingly, we will show in Section 5 that, if we consider more types (e.g. high, medium, low), then overproducing to signal capacity can be profitable for the high type even in stationary settings. Thus, it can be profitable to have capacity ambiguity but then overproduce to signal capacity under fairly general conditions.

5. EXTENSIONS

Our model in the previous section had only two types of capacity levels and only one firm whose capacity information was private. To verify robustness of the insights from our model, we have extended the basic model in several directions. First, we consider both firms having private information about their own capacities. We show that, in such settings, due to two-sided information asymmetry, there exist partial

Table 4. Example that the Firm 2 benefits from information asymmetry ($a^1 = 30, a^2 = 90, b^1 = b^2 = 1, c_1 = c_2 = 0, \overline{K} = 25, \underline{K} = 5, K_2 = 35, \mu = 0.5$).

	Asymmetric information					Full information				
	Q_1^1	Q_1^2	π^1	π^2	Π	Q_1^1	Q_1^2	π^1	π^2	Π
Firm 1 ($\theta = h$)	9.286	25	86.224	812.5	898.724	10	25	100	812.5	912.5
Firm 1 ($\theta = l$)	5	5	67.857	250	317.857	5	5	62.5	250	312.5
Firm 2	11.429	32.5 (h) 35 (l)	130.612	1403.125	1533.737	10 (h) 12.5 (l)	32.5 (h) 35 (l)	128.125	1403.125	1531.25

pooling and partial separating equilibria, where, the high and low types of one firm may produce the same quantity, whereas for the other firm, the equilibrium is a separating equilibrium. Otherwise, the main insights remain similar (i.e., it may still be optimal for firms to overproduce to signal their capacity, but firms may benefit from capacity ambiguity despite having to overproduce to then reveal their capacity).

In our second extension, we considered the case where the number of types equals three. The reason this is interesting is that we are able to show that even when the price function is stationary from Period 1 to 2, the high-type firm may have an incentive to overproduce to reveal its type. Recall that this was not the case for two types when price was stationary. Thus, with more than two types, even under stationary demand, a firm may benefit from strategically revealing its capacity and the incentive to do so depends on the distribution of beliefs on its capacity. We conjecture that increasing number of capacity types or technology allowing more capacity choices, create additional incentives to differentiate.

Our model to this point assumes that firms start with fixed capacities (and their capacity level information is private to them) and they have to make production decisions. An interesting question is whether our conclusions are robust if the firms can choose their capacity. One may initially believe that our results only occur because firms ended up with their capacity levels, and that if they were choosing what capacity level to invest in, they would never choose capacity levels where they would have to spend extra resources to then signal their capacity. Interestingly, this is not the case. In Section 5.3, we consider the case where one of the firms, for example, Firm 1 is choosing its capacity level but the competing firm (Firm 2) has uncertainty about Firm 1's capacity investment cost. We show that this situation can still result in Firm 1 having to overproduce to signal its capacity level. That is, even if firms choose their capacity level optimally, under capacity investment cost ambiguity, this can still result in firms having to expend resources to signal their chosen capacity level. Finally, in Section 5.4, we consider the scenario where a firm may have information asymmetry regarding its competitor's cost instead of its capacity and show that this results in very similar insights.

In the following, we summarize the main results we obtained for each of these extensions but omit the detailed proofs of the results, which can be seen in Ye et al. [25].

5.1. Two-Sided Information Asymmetry

Suppose that the two Firms, 1 and 2, have private information about their own capacities, K_1 and K_2. For each firm, we continue to consider two types with the same, high $\overline{K}$ and low $\underline{K}$ capacities, $\overline{K} > \underline{K}$. Firms' beliefs may be asymmetric. At the beginning of the first period, Firm 1's belief about Firm 2 being high type is $\lambda \in [0, 1]$, and Firm 2's belief about Firm 1 being high type is $\mu \in [0, 1]$. Without loss of generality, it is assumed that $c_1 \geq c_2$. To avoid trivial solutions, we assume that $a - 2c_1 + c_2 \geq 0$ (which implies $a - 2c_2 + c_1 \geq 0$). Denote by $Q^*_{i\theta}(\lambda, \mu)$ and $\pi^*_{i\theta}(\lambda, \mu)$ the equilibrium quantity and profit for firm i of type θ, for $i = 1, 2$ and $\theta = h, l$, in one-period game.

Because each firm can be of any of two types, there exist three types of equilibria: (1) pooling equilibrium, where both firms produce the same quantity independent of their types, (2) separating equilibrium, where each firm's production quantity depends on its type, and (3) partial pooling and partial separating equilibrium, where one firm produces the same quantity regardless of its type, whereas the other firm produces different quantities depending on type. As in the case of the one-sided asymmetric information model, we focus on the case where the second-period market has higher potential $\frac{a^1-2c_2+c_1}{3b^1} \leq \frac{a^2-2c_2+c_1}{3b^2}$. We have the following results for the model with two-sided information asymmetry (the equilibrium partition is shown in Fig. 4).

THEOREM 4:

a. If $\underline{K} \geq \frac{a^2-2c_2+c_1}{3b^2}$ or $\{\overline{K} \leq \frac{a^2-c_1}{3b^2}$ and $\underline{K} \geq \frac{a^1-2c_2+c_1}{3b^1}\}$, there exists a unique pooling equilibrium.

b. If $\{\overline{K} \leq \frac{a^2-2c_2+c_1}{3b^2}$ and $\underline{K} < \max\{\frac{(2-\lambda)a^1-2c_1+\lambda c_2}{(2-\lambda)3b^1}, \frac{a^1-c_1}{(3-\lambda)b^1} - \frac{\lambda}{3-\lambda}\overline{K}\}\}$ or $\{\overline{K} > \frac{a^2-2c_2+c_1}{3b^2}$ and $\underline{K} < \frac{a^2-2c_1+c_2}{3b^2}\}$, there exists a unique separating equilibrium.

- b1. If $\underline{K} < \max\{\frac{(2-\lambda)a^1-2c_1+\lambda c_2}{(2-\lambda)3b^1}, \frac{a^1-c_1}{(3-\lambda)b^1} - \frac{\lambda}{3-\lambda}\overline{K}\}$, both firms produce the myopic equilibrium in the first period;
- b2. If $\overline{K} > \frac{a^2-2c_2+c_1}{3b^2}$ and $\frac{(2-\lambda)a^1-2c_1+\lambda c_2}{(2-\lambda)3b^1} < \underline{K} < \min\{\frac{a^1-2c_2+c_1}{3b^1}, \frac{a^2-2c_1+c_2}{3b^2}\}$, the high type of Firm 1 overproduces in the first period (compared to the myopic solution) to differentiate itself;
- b3. If $\overline{K} > \frac{a^2-2c_2+c_1}{3b^2}$ and $\frac{a^1-2c_2+c_1}{3b^1} < \underline{K} < \max\{\frac{a^1-2c_2+c_1}{3b^1}, \frac{a^2-2c_1+c_2}{3b^2}\}$, the high type of both firms overproduces in the first period to differentiate themselves.

c. Otherwise, there exists a partial pooling and partial separating equilibrium where firm 1 produces the same quantity independent of its type, whereas Firm 2 produces different quantity depending on its type.

- c1. If $\underline{K} \leq \frac{a^1-2c_2+c_1}{3b^1}$, both firms produce myopic equilibrium in the first period;
- c2. If $\underline{K} > \frac{a^1-2c_2+c_1}{3b^1}$, the high type of Firm 2 overproduces in the first period to differentiate itself.

The results of Theorem 4 are illustrated in Fig. 4a. The complication here is that because there are two firms both with

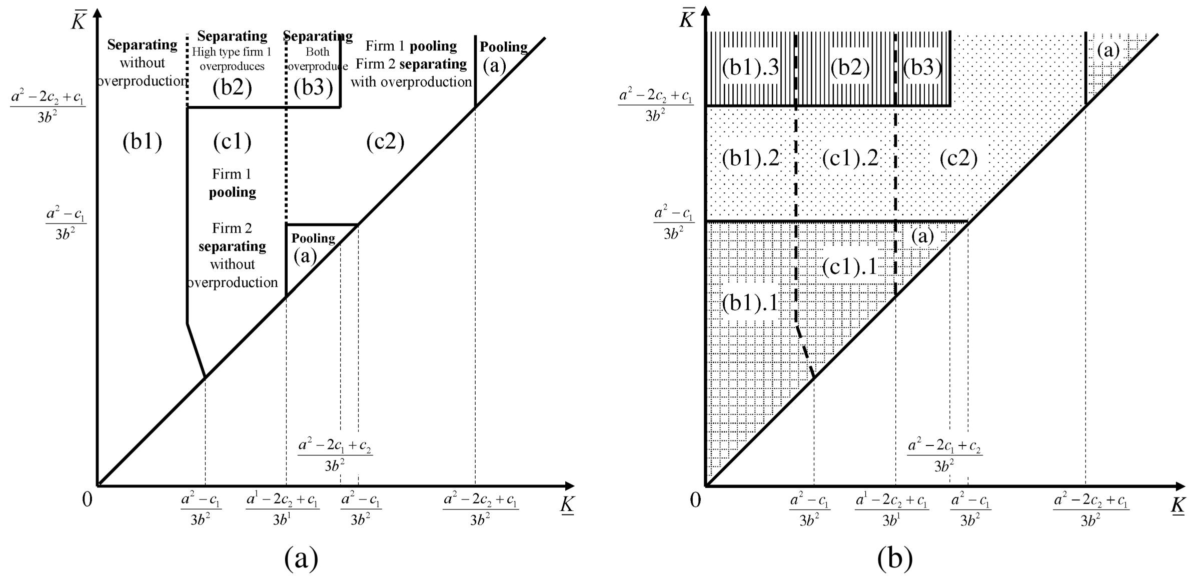

Figure 4. Equilibrium partition for the model with two-sided information asymmetry (a) Effects of capacity information in the second period and (b) Partition of equilibrium production decisions in the first period.

private capacity information, we could have situations where one firm could try to differentiate itself whereas the other does not. To understand the derivation of the results shown in Fig. 4a, we divide the region (b1) into three parts, and (c1) into two sub-regions, as shown in Fig. 4b. In regions (b1).1, (c1).1, and (a), the firms' beliefs about the competitor's capacity do not affect their production decisions in the second period. This is because either capacity is ample (specifically, even the low-capacity limit $\underline{K}$ is higher than $\frac{a^2-2c_2+c_1}{3b^2}$), capacity limits have no impact on production, or when capacity is tight (specifically, even the high-capacity limit $\overline{K}$ is less than $\frac{a^2-c_1}{3b^2}$), both firms use full capacity regardless of their types. In the other regions, firms' capacity information influences their production decisions. However, the effects differ depending on firms' production costs. Firm 2 earns additional profit by differentiating itself successfully in all regions ((b1).2, (b1).3, (b2), (b3), (c1).1, and (c2)), whereas Firm 1 cares about Firm 2's belief on its type only in regions (b1).3, (b2), and (b3) because it has higher production cost than Firm 2. That is, incentives to differentiate by overproducing are different for the two firms because of their different costs.

Because in regions (b1).1, (c1).1, and (a), the capacity information does not influence firms' production decisions in the second period, each player chooses its myopic solution in the first period. As $\underline{K}$ increases across these regions in Fig. 4b, the type of equilibrium changes from full separating equilibrium to partial pooling and partial separating one to full pooling one. Even though the outcome may be separating or pooling, the firms achieve these by just following the myopic solutions.

In regions (b1).2, (c1).2, and (c2), Firm 2 cares about capacity information in the second period, whereas Firm 1 produces myopic solution in the first period regardless of its type. The high type of Firm 2 overproduces when the high-type's myopic solution can be mimicked by the low type. If $\underline{K}$ is low enough (which corresponds to region (b1).2 in Fig. 4(b)), myopic solution for the high type separates naturally. Therefore, both firms play myopically which results in a fully separating equilibrium. In region (c1).2, for Firm 2 the myopic solution is between $\underline{K}$ and $\overline{K}$. Thus, the high type does not need to worry about being confused with the low type and can safely produce the myopic solution which naturally separates the two types. On the other hand, in region (c2), the myopic quantity for Firm 2 is less than $\underline{K}$. Therefore, the high type of Firm 2 is afraid that firm 1 will not be able to differentiate between the two types if both high and low type produce the myopic quantity. Thus, the high type overproduces to differentiate itself. The explanation for all the other regions follows the same logic.

The result in Theorem 4 shows that the main intuition gained in Section 4, where only one firm has private capacity information, is essentially carried over to the case where both firms own private information. Once again, a growing market may force a high-capacity firm to spend extra by overproducing to reveal its type in the first period; whereas a low-capacity firm essentially prefers the information asymmetry to remain intact. However, even a high-capacity firm may sometimes benefit from information asymmetry.

The next section extends our results to the case with three capacity levels.

5.2. Three Capacity Types

For simplicity, in this case, we only consider the stationary price function and assume that the price function is $p = a - b(Q_1 + Q_2)$ where Q_1 and Q_2 are production quantities of Firm 1 and 2. An interesting result with three types is that, unlike the case with two types, even under stationary price/demand, firms may choose to signal their capacity by overproducing.

As in our base model, we assume that Firm 1 has private information about its own capacity K_1, and Firm 2's capacity is common knowledge. Firm 2 knows that its rival's capacity can be high K_h, medium K_m, or low K_l, with $K_h > K_m > K_l$. At the beginning of the first period, Firm 2 believes that probabilities of K_h, K_m, and K_l are p_h, p_m, and p_l, respectively, with $p_h + p_m + p_l = 1$. In the first period two firms simultaneously produce Q_1^1 and Q_2^1. In the second period, Firm 2 updates its belief about Firm 1's type, based on Firm 1's production quantity in the first period, and the two firms produce Q_1^2 and Q_2^2. For notational simplicity, the production costs for both firms are zero.

Given that Firm 1 may be of any of three types, high, medium, or low, there exist different types of equilibria: (1) pooling equilibrium, where three types of Firm 1 produce the same quantity at equilibrium, (2) separating equilibrium, where the three types of each firm produce different quantities at equilibrium, and (3) partial pooling and partial separating equilibrium, where two of three types produce the same quantity, and the other type produces different quantity. To avoid trivial solutions, we assume $p_h, p_m, p_l \in (0, 1)$. For the three-type model, we have the following results.

THEOREM 5:

a. If $K_l \geq \frac{a}{3b}$, there exists a unique pooling equilibrium. That is, in the first period, three types produce the same quantity.
b. If $K_l < \frac{a}{3b}$ and $(3 + p_l)K_m - p_l K_l < \frac{a}{b}$, there exists a unique separating equilibrium. That is, in the first period, three types produce different quantities. Moreover, each type produces myopic equilibrium quantity.
c. If $K_l < \frac{a}{3b}$ and $(3 + p_l)K_m - p_l K_l \geq \frac{a}{b}$, then
   - c1. If $K_m \geq \frac{a}{3b}$, there exists unique partial pooling equilibrium: in the first period, high and medium types produce the same quantity, and low type produce different quantity.
   - c2. If $K_m < \frac{a}{3b}$, there exists unique separating equilibrium: in the first period, three types produce different quantities. Moreover, the high type deviates from its myopic equilibrium quantity, that is, it overproduces to signal its type in the first period; while the medium and low type produce myopic equilibrium quantities in the first period.

Recall that in Section 4.1, we showed that, when demand is stationary (modeled as a stationary price function) and there are only two types of players, low and high, each type produces its myopic one-period equilibrium in the first period. That is, neither type worries about Firm 2's belief updating in the second period. However, as we show in the above theorem, in the three-type case, there exist some situations where this indifference does not hold anymore. When $K_m < \frac{a}{3b}$ and $(3 + p_l)K_m p_l K_l \geq \frac{a}{b}$, the high type has an incentive to signal its type in the first period by overproducing even when demand is stationary. This essentially happens when the first period myopic solution for the high type falls between the low-type and the medium-type's capacities. Thus, the low type can still not imitate the high type in the first period, but the medium type can.

It is interesting to consider why the high-type firm facing stationary demand never overproduces to differentiate itself in the case when there are two types, but it may have to overproduce when there are more than two types. Consider the case with two types, and assume that K_l is fairly low so that it is optimal for the low-type firm to produce at its capacity in both periods, whereas the high-type's myopic quantities are above K_l. In this case, if the high-type firm produced its myopic quantity in the first period, this would naturally signal to Firm 2 that it is high type. Of course, the greater Firm 2's belief that it is facing a low-type firm, the more the high-type firm has to lower its production quantity in the first period, but this never ends up below K_l and thus it is ensured that Firm 2 perfectly knows it is facing a high-type firm in the second period. Now, consider the case with three types. Assume once again that K_l is very low. The problem now is that if Firm 2 has a very strong belief that it is facing a low-type firm, it is likely to produce a large quantity in the first period, and the greater the value of p_l, the more the high-type firm has to lower its first period production quantity to account for Firm 2's belief. This myopic quantity in the first period may now be lower than K_m (because of the high value of p_l), whereas the high-type firm may want to produce at a level higher than K_m in the second period. (Notice that the condition $(3 + p_l)K_m - p_l K_l \geq \frac{a}{b}$ is very likely to be satisfied with very high values of p_l and very low values of K_l). For Firm 1, it may be beneficial to signal to Firm 2 that it is indeed a high-type firm, and not a medium type. Such signaling is possible by increasing production to a level that is higher than the myopic level of the high type and which would not be economical for the middle type. Thus, in this case, Firm 2's very high belief that it is facing a low-type firm may drag Firm 1's myopic production quantity in the first period to below K_m, where unlike in the two-period case, it is no longer differentiated as a high-type firm and this

Table 5. Example that Firm 1 with lower-capacity cost invests in more capacity and overproduces to signal its capacity level ($a^1 = 6, a^2 = 18, b^1 = b^2 = 1, c_1 = c_2 = 0, \mu = 0.5$).

Parameters			Capacity decisions		Production decisions		Profits	
K_2	C_l	C_h	$\overline{K}$	$\underline{K}$	Q_{1h}	Q_{1l}	Π_{1h}	Π_{1l}
6.1	1	2	6	5	2.825	2.118	33.985	28.985
6.1	1	3	6	4.5	2.783	2.112	34.010	24.260
6.1	1	4	6	4	2.738	2.105	34.033	20.033
10	1	2	6	5.8	2.889	2.127	33.944	28.304
10	1	3	6	5.3	3.589	2.227	33.105	22.715
10	1	4	6	4.6	4.094	2.299	32.066	17.706

may force it overproduce even when it is facing a stationary demand.

5.3. Asymmetric Information on Capacity Investment Cost

In the previous cases, we assumed that firms have fixed capacities when they start the game and that they do not choose their capacity levels. This raises the natural question whether the results we observed are an artifact of this assumption. That is, if a firm were choosing its capacity level optimally, would it actually choose a level that requires expanding even more resources to then signal capacity to competitors? That is, it seems possible that firms would not choose capacity levels which would then require them to spend even more resources to signal to competitors. To address this question, we extend the base model by assuming the information asymmetry about the capacity investment cost. We assume that Firm 2 is an incumbent with publicly known capacity K_2 in the market, and Firm 1 is an entrant who needs to decide how much capacity to acquire. We make the reasonable assumption that capacity building takes significantly longer than the production period durations and, therefore, the firm will make one capacity decision and then have to live with it for its production decisions (e.g., Advanced Micro Devices (AMD) or Intel cannot change their capacity every month but typically make a capacity decision for one product generation lifecycle). Thus, in our model, Firm 1 first invests in capacity and, then, competes with the incumbent Firm 2 in quantity over two periods. (As before, we can extend this to two-sided information asymmetry, e.g., both firms deciding on their capacity levels with uncertain capacity investment costs without changing the main insights).

We denote Firm 1's unit capacity investment cost as C_K and this is private information for Firm 1. Despite not knowing Firm 1's capacity investment cost, Firm 2 has some initial beliefs on it. For simplicity, we assume that Firm 2 believes Firm 1's unit capacity cost can be either low or high. Specifically, Firm 2 believes $C_K = C_l$ with probability $\mu \in [0, 1]$, and $C_K = C_h$ with probability $1 - \mu$ where $C_l < C_h$. Due to the information asymmetry on capacity investment cost, Firm 2 is uncertain on his competitor's capacity before playing the two-stage capacitated Cournot competition. Intuitively, the entrant Firm 1 with low-capacity investment cost is able to build higher capacity than if its capacity investment cost were high. Denote $\overline{K}$ and $\underline{K}$ as the capacity levels of Firm 1 with low and high-capacity investment cost, respectively. Because nothing can be inferred in the stage of capacity investment, Firm 2 believes Firm 1's capacity $K_1 = \overline{K}$, with probability $\mu \in [0, 1]$, and $K_1 = \underline{K}$, with probability $1-\mu$. To determine their equilibrium capacity investment level, the entrant Firm 1 of each type needs to consider the trade-off between its capacity investment cost and the payoffs from the following two-stage Cournot game.

For $i = h, l$, let $\Pi_{1h}(\overline{K}, \underline{K})$ be the payoff function of Firm 1 of type i in the two-period Cournot competition given that high- and low-type firm 1 has capacity $\overline{K}$ and $\underline{K}$ with $\overline{K} > \underline{K}$. Thus, the total profit function for firm 1 of type i is

$$V_{1h}(\overline{K}, \underline{K}) = -C_l\overline{K} + \Pi_{1h}(\overline{K}, \underline{K})$$
$$V_{1l}(\overline{K}, \underline{K}) = -C_h\underline{K} + \Pi_{1l}(\overline{K}, \underline{K})$$

Based on the equilibrium production quantities of both types in each period, given in Theorems 1–3, $\Pi_{1i}(\overline{K}, \underline{K})$ can be easily derived. To avoid extensive derivations, we will not provide the derivations on capacity investment levels for Firm 1 and production quantities for two periods for both firms. Instead, some numerical examples are provided in Table 5 to show that in some cases there exist pure strategy equilibrium capacity decisions for different types where the entrant firm with low-capacity cost may overproduce in the first Cournot competition to signal its capacity type.

In all of the examples in Table 5, the high-type overproduces (notice that the low type could replicate the high-type's production levels as its capacity level is sufficient to produce at the high-type's level but it is in fact economical for the high-type firm to overproduce and not for the low type). Like in our base model, these examples are for a case with growing demand over time. Notice that this is in fact very reasonable, because it is probably the growing demand that would attract entrants in the first place. Thus, it is interesting that even when capacity decisions are endogenized, cases where firms have to overproduce to signal their capacity or where they benefit from information asymmetry can occur.

Intuitively when the capacity decision is endogenized and the entrant firm's capacity investment cost is private information, the low-capacity type entrant firm (i.e., the firm with high-capacity investment cost) always benefits from the uncertainty of its capacity investment cost. However, similar to the case where there is information asymmetry on the exogenous capacity level (analyzed before), how information asymmetry affects the performance of high-capacity type entrant firm is subtle. We are able to numerically show that the entrant Firm 1 with low-capacity investment cost may be better off when the incumbent firm is uncertain about its capacity cost type. For instance, in Table 5, if Firm 1 is known as high-capacity type (i.e., its capacity cost $C_l = 1$ is known), it is easy to show that Firm 1's optimal capacity investment level is 6, and its maximum total profit (including the capacity investment cost) is 34. In the second and third row of Table 5, the expected profits of the high-capacity type entrant Firm 1 are higher than in the full-information case.

5.4. Asymmetric Information on Production Cost

This article has focused on information asymmetry on capacity and whether firms have incentives to signal their capacity levels in certain cases by overproducing. It is of course plausible to argue that firms may have information asymmetries about other parameters of competitive interest, such as production costs (we showed in the previous section that asymmetry on capacity investment cost results in exactly the same insights as asymmetry on capacity levels).

It is easy to show that if firms have production cost information asymmetry, the same type of incentives to overproduce arise again. Interestingly, in the simplest case where both firms have ample capacity, we can show that the "high-type" firm (i.e., the firm with the lower-production cost) can always differentiate itself (i.e, there is no pooling equilibrium.) To see this, suppose that Firm 1 has private information on his production cost, c_1, whereas Firm 2's production cost c_2 is public information. As before, we assume that two values are possible for Firm 1's production cost, $c_1 = c_l$ or c_h with $c_l < c_h$. Firm 1 with low-production cost c_l is referred to as a high-type firm, and c_h as a low type. The firms play two-period Cournot competition, and price functions for each period $t = 1, 2$ are $p^t = a^t - b^t(Q_1^t + Q_2^t)$ where Q_i^t are firm i's production quantity in period t. The following theorem characterizes firms' two-period equilibrium decisions.

THEOREM 6:

a. If $a^2 \leq \frac{9b^2+7b^1}{4b^1}c_h - \frac{9b^2-b^1}{4b^1}c_l - c_2$ i.e., $\pi_{1l}^1(Q_{1l}^{1*}, Q_2^{1*}) - \pi_{1l}^1(Q_{1h}^{1*}, Q_2^{1*}) \geq \pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)$, there exists a unique separating and myopic equilibrium. At equilibrium, in Period 1 each firm produces

$$Q_{1h}^{1*} = \frac{2a^1 + 2c_2 - (3+\mu)\,c_l - (1-\mu)\,c_h}{6b^1},$$
$$Q_{1l}^{1*} = \frac{2a^1 + 2c_2 - (4-\mu)\,c_h - \mu c_l}{6b^1},$$
$$Q_2^{1*} = \frac{a^1 - 2c_2 + \mu c_l + (1-\mu)\,c_h}{3b^1}.$$

b. If $a^2 > \frac{9b^2+7b^1}{4b^1}c_h - \frac{9b^2-b^1}{4b^1}c_l - c_2$ (i.e., $\pi_{1l}^1(Q_{1l}^{1*}, Q_2^{1*}) - \pi_{1l}^1(Q_{1h}^{1*}, Q_2^{1*}) < \pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)$), there exists a unique separating equilibrium. At equilibrium, in Period 1 each firm produces

$$Q_{1h}^{1*} = \frac{a^1 - 2c_h + c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)}{b^1}},$$
$$Q_{1l}^{1*} = \frac{a^1 - 2c_h + c_2}{3b^1} + \frac{\mu}{3}\sqrt{\frac{\pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)}{b^1}},$$
$$Q_2^{1*} = \frac{a^1 - 2c_2 + c_h}{3b^1} - \frac{2\mu}{3}\sqrt{\frac{\pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)}{b^1}}.$$

When firms' production costs are private information and their capacities are ample, there is always a unique separating equilibrium in Period 1 which reveals Firm 1's type at Period 2. As shown in Theorem 6(a), when the market size in the second period a^2 is low, all firms choose the myopic production quantities in the first period. If a^2 is low, it is too costly for the low type to produce the high-type's myopic quantity in Period 1 because the extra profit for the low type by successfully pretending to be a high type in Period 2 cannot recoup his extra cost to mimic the high-type's myopic quantity in Period 1. The high type, therefore, ignores the low-type's decision, and chooses his one-period (i.e., myopic) quantity decision in Period 1. As shown in Theorem 6(b), however, if the market size in the second period a^2 is high, the high type Firm 1 will not produce the myopic quantity in Period 1 because if he does so, the low type will produce the same amount and get higher profit in Period 2 by pretending to be a high type than the extra cost incurred in Period 1 by mimicking the high type. Therefore, in this scenario the high type will overproduce to signal his type. The quantity of overproduction of the high type is determined by the incentive of the low type to mimic the high-type's decision, $\pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)$.

Thus, production cost asymmetry may lead to incentives to overproduce just like capacity information asymmetry. Of course, it is plausible to think that firms may have asymmetry of information on both costs and capacities. However, problems with more than one dimension of information asymmetry are very difficult to analyze (see, e.g., Kostamis and Duenyas [18] which is one of the few papers that looks at such a problem in a very different setting). Thus, we leave this natural extension to further research.

6. CONCLUSIONS

In this article, we addressed a dynamic Cournot game, where two firms compete across two periods by simultaneously supplying production quantities to market. We considered situations where the capacity information is private and the firms update their beliefs about their rival's capacity after observing its output in the first period. We studied how the information asymmetry about capacity affects firms' production decisions. We showed that there exist pooling or separating equilibria which depend on the initial belief and capacity levels for each type and each firm. With two types of capacity levels, when demand is stationary or decreasing across two periods, each firm produces its myopic optimal quantity in the first period. When, however, demand is increasing over the two periods, it is possible for the firm with high capacity to deviate from its myopic optimal decision and sacrifice part of its short-run profits to differentiate its type from the firm with low capacity, and thus maximize the total profits over two periods. Furthermore, we showed that the information asymmetry in capacity always favors the firm with low capacity. However, even the firm with high capacity can benefit from information asymmetry. Also, the firm who does not know competitor's capacity may benefit from this information asymmetry.

We extended the basic model to include two-sided information asymmetry, three capacity types and also analyzed the case where capacity decisions can be endogenized. We showed that the main insights from the one-sided information asymmetry case continue to hold with two-sided asymmetry. We also showed that, when the number of types is greater than two, the high type may have a need to differentiate itself in the first period even under stationary demand. Finally, our main insights continue to hold even when capacity decisions are endogenized or if information asymmetry exists regarding production cost rather production capacity.

Our results indicate that there is a good reason why many firms in practice take care to hide their capacity level information, whereas others attempt to reveal it. Our model suggests that low-capacity firms benefit from ambiguity and have the highest incentive to ensure their capacity levels do not become public. For high-capacity firms, the effect is more subtle. Sometimes, the firm may even be willing to overproduce to signal its capacity and yet in some of those cases, the firm may recoup the costs of overproduction in future periods so that information asymmetry may still be advantageous. In fact, in some cases, a firm may even prefer not to know its competitors' capacity. Our results indicate that firms need to take such complications into account in deciding to influence beliefs about their capacity.

Our model is highly stylized in the context of two competing firms. Still, we believe it does provide a good understanding of the essential tradeoffs to be considered in dealing with capacity information asymmetry. Further research should focus on operationalizing the insights provided in this article with more general capacity functions than the ones considered here and also consider other supply chain contexts (e.g., an OEM buying from a supplier where the supplier or OEM's capacity may be ambiguous).

APPENDIX

PROOF OF LEMMA 1: From the Eq. (1)–(3), the equilibrium solutions depend on the capacity parameters $\overline{K}, \underline{K}$ and K_2, as well as the belief μ. We solve the equilibrium solutions for the cases $K_2 \le \frac{a-2c_2+c_1}{3b}$ and $K_2 > \frac{a-2c_2+c_1}{3b}$, separately. □

CASE 1. When $K_2 \le \frac{a-2c_2+c_1}{3b}$, the equilibrium quantities and profits for high-type Firm 1, low-type Firm 1, and Firm 2 are obtained for the following three regions:

1. If $\overline{K} \le \frac{a-c_1}{2b} - \frac{1}{2}K_2$, then $Q^*_{1h}(\mu) = \overline{K}$, $Q^*_{1l}(\mu) = \underline{K}$, and $Q^*_2(\mu) = K_2$. The equilibrium profits are

$$\begin{aligned}\pi^*_{1h}(\mu) &= [a - c_1 - b(\overline{K} + K_2)]\overline{K}\\ \pi^*_{1l}(\mu) &= [a - c_1 - b(\underline{K} + K_2)]\underline{K}\\ \pi^*_2(\mu) &= [a - c_2 - bK_2 - \mu b\overline{K} - (1-\mu)b\underline{K}]K_2.\end{aligned}$$

2. If $\overline{K} \ge \frac{a-c_1}{2b} - \frac{1}{2}K_2$ and $\underline{K} \le \frac{a-c_1}{2b} - \frac{1}{2}K_2$, then $Q^*_{1h}(\mu) = \frac{a-c_1}{2b} - \frac{1}{2}K_2$, $Q^*_{1l}(\mu) = \underline{K}$, and $Q^*_2(\mu) = K_2$. The equilibrium profits are

$$\begin{aligned}\pi^*_{1h}(\mu) &= \frac{b}{4}\left(\frac{a-c_1}{b} - K_2\right)^2\\ \pi^*_{1l}(\mu) &= [a - c_1 - b(\underline{K} + K_2)]\underline{K}\\ \pi^*_2(\mu) &= \left[\frac{(2-\mu)a-2c_2+\mu c_1}{2} - \frac{2-\mu}{2}bK_2 - (1-\mu)b\underline{K}\right]K_2.\end{aligned}$$

3. If $\underline{K} \ge \frac{a-c_1}{2b} - \frac{1}{2}K_2$, then $Q^*_{1h}(\mu) = \frac{a-c_1}{2b} - \frac{1}{2}K_2$, $Q^*_{1l}(\mu) = \frac{a-c_1}{2b} - \frac{1}{2}K_2$, and $Q^*_2(\mu) = K_2$. The equilibrium profits are

$$\begin{aligned}\pi^*_{1h}(\mu) &= \frac{1}{4}b\left(\frac{a-c_1}{b} - K_2\right)^2\\ \pi^*_{1l}(\mu) &= \frac{1}{4}b\left(\frac{a-c_1}{b} - K_2\right)^2\\ \pi^*_2(\mu) &= \frac{1}{2}bK_2\left(\frac{a-2c_2+c_1}{b} - K_2\right).\end{aligned}$$

Figure 5 illustrates the equilibrium partition for the case $K_2 \le \frac{a-2c_2+c_1}{3b}$. The results show that, for $K_2 \le \frac{a-2c_2+c_1}{3b}$, the equilibrium solutions for both types of Firm 1 are independent of belief μ.

CASE 2. When $K_2 > \frac{a-2c_2+c_1}{3b}$, the equilibrium quantities and profits for high-type Firm 1, low-type Firm 1, and Firm 2 are obtained for the following five regions:

1. If $\overline{K} \le \frac{a-c_1}{2b} - \frac{1}{2}K_2$ and $\mu\overline{K} + (1-\mu)\underline{K} \le \frac{a-c_2}{b} - 2K_2$, then $Q^*_{1h}(\mu) = \overline{K}$, $Q^*_{1l}(\mu) = \underline{K}$, and $Q^*_2(\mu) = K_2$. The equilibrium profits are

$$\begin{aligned}\pi^*_{1h}(\mu) &= [a - c_1 - b(\overline{K} + K_2)]\overline{K}\\ \pi^*_{1l}(\mu) &= [a - c_1 - b(\underline{K} + K_2)]\underline{K}\\ \pi^*_2(\mu) &= [a - c_2 - bK_2 - \mu b\overline{K} - (1-\mu)b\underline{K}]K_2.\end{aligned}$$

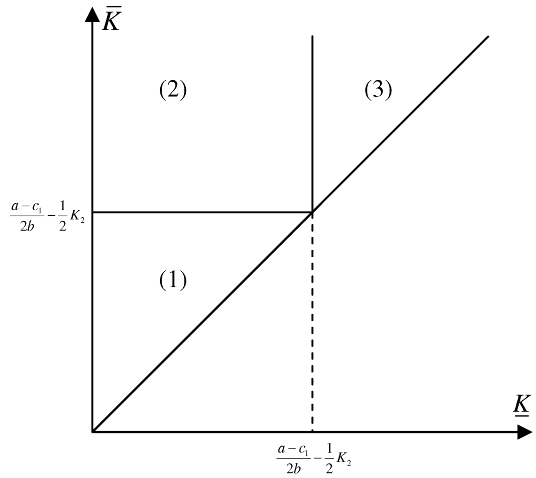

Figure 5. Equilibrium partition when $K_2 \le \frac{a-2c_2+c_1}{3b}$.

2. If $\overline{K} > \frac{a-c_1}{2b} - \frac{1}{2}K_2$ and $\underline{K} \le \frac{(2-\mu)a-2c_2+\mu c_1}{(2-2\mu)b} - \frac{4-\mu}{2-2\mu}K_2$, then $Q^*_{1h}(\mu) = \frac{a-c_1}{2b} - \frac{1}{2}K_2$, $Q^*_{1l}(\mu) = \underline{K}$, and $Q^*_2(\mu) = K_2$. The equilibrium profits are

$$\pi^*_{1h}(\mu) = \tfrac{1}{4}b(\tfrac{a-c_1}{b} - K_2)^2$$
$$\pi^*_{1l}(\mu) = [a - c_1 - b(\underline{K} + K_2)]\underline{K}$$
$$\pi^*_2(\mu) = [\tfrac{(2-\mu)a-2c_2+\mu c_1}{2} - \tfrac{2-\mu}{2}bK_2 - (1-\mu)b\underline{K}]K_2.$$

3. If $(4-\mu)\overline{K} - (1-\mu)\underline{K} \le \frac{a-2c_1+c_2}{b}$ and $\mu\overline{K} + (1-\mu)\underline{K} > \frac{a-c_2}{b} - 2K_2$, then $Q^*_{1h}(\mu) = \overline{K}$, $Q^*_{1l}(\mu) = \underline{K}$,and $Q^*_2(\mu) = \frac{a-c_2}{2b} - \frac{1}{2}[\mu\overline{K} + (1-\mu)\underline{K}]$. The equilibrium profits are

$$\pi^*_{1h}(\mu) = \left(\tfrac{a-2c_1+c_2}{2} - \tfrac{2-\mu}{2}b\overline{K} + \tfrac{1-\mu}{2}b\underline{K}\right)\overline{K}$$
$$\pi^*_{1l}(\mu) = \left(\tfrac{a-2c_1+c_2}{2} + \tfrac{\mu}{2}b\overline{K} - \tfrac{1+\mu}{2}b\underline{K}\right)\underline{K}$$
$$\pi^*_2(\mu) = \tfrac{1}{4}b\left[\tfrac{a-c_2}{b} - \mu\overline{K} - (1-\mu)\underline{K}\right]^2.$$

4. If $(4-\mu)\overline{K} - (1-\mu)\underline{K} > \frac{a-2c_1+c_2}{b}$ and $\frac{(2-\mu)a-2c_2+\mu c_1}{(2-2\mu)b} - \frac{4-\mu}{2-2\mu}K_2 < \underline{K} \le \frac{a-2c_1+c_2}{3b}$, then $Q^*_{1h}(\mu) = \frac{a-2c_1+c_2}{(4-\mu)b} + \frac{1-\mu}{4-\mu}\underline{K}$, $Q^*_{1l}(\mu) = \underline{K}$, and $Q^*_2(\mu) = \frac{(2-\mu)a-2c_2+\mu c_1}{(4-\mu)b} - \frac{2-2\mu}{4-\mu}\underline{K}$. The equilibrium profits are

$$\pi^*_{1h}(\mu) = b\left[\tfrac{a-2c_1+c_2}{(4-\mu)b} + \tfrac{1-\mu}{4-\mu}\underline{K}\right]^2$$
$$\pi^*_{1l}(\mu) = b\underline{K}\left[\tfrac{2(a-2c_1+c_2)}{(4-\mu)b} - \tfrac{2+\mu}{4-\mu}\underline{K}\right]$$
$$\pi^*_2(\mu) = b\left[\tfrac{(2-\mu)a-2c_2+\mu c_1}{(4-\mu)b} - \tfrac{2-2\mu}{4-\mu}\underline{K}\right]^2.$$

5. If $\underline{K} > \frac{a-2c_1+c_2}{3b}$, then $Q^*_{1h}(\mu) = Q^*_{1l}(\mu) = \frac{a-2c_1+c_2}{3b}$, and $Q^*_2(\mu) = \frac{a-2c_2+c_1}{3b}$. The equilibrium profits

$$\pi^*_{1h}(\mu) = \pi^*_{1l}(\mu) = \tfrac{(a-2c_1+c_2)^2}{9b}$$
$$\pi^*_2(\mu) = \tfrac{(a-2c_2+c_1)^2}{9b}.$$

The five regions are illustrated in Fig. 6. Note that as K_2 increases, the boundary line of region (4), $\frac{(2-\mu)a-2c_2+\mu c_1}{(2-2\mu)b} - \frac{4-\mu}{2-2\mu}K_2$, will move gradually to the left. When $\frac{a-2c_2+c_1}{3b} < K_2 \le \frac{(2-\mu)a-2c_2+\mu c_1}{(4-\mu)b}$, all five regions are nonempty; when $\frac{(2-\mu)a-2c_2+\mu c_1}{(4-\mu)b} < K_2 \le \frac{a-c_2}{2b}$, region (2) disappears; and when $K_2 > \frac{a-c_2}{2b}$, both region (1) and (2) disappear.

In summary, the equilibrium quantities and profits of high- and low-type Firm 1 are functions of belief μ (moreover, it is easy to verify they are strictly increasing in μ) in region (3) and (4) of case 2, that is, $K_2 > \frac{a-2c_2+c_1}{3b}$ and $\mu\overline{K} + (1-\mu)\underline{K} > \frac{a-c_2}{b} - 2K_2$ and $\frac{(2-\mu)a-2c_2+\mu c_1}{(2-2\mu)b} - \frac{4-\mu}{2-2\mu}K_2 < \underline{K} \le \frac{a-2c_1+c_2}{3b}$; otherwise, their equilibrium quantities and profits are independent of μ.

PROOF OF THEOREM 1. We start by introducing a technical lemma which we will use in the proof of Theorem 1. □

LEMMA A1: If $\frac{a^1-2c_1+c_2}{3b^1} \ge \frac{a^2-2c_1+c_2}{3b^2}$ and $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$ and $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$, then there can be no separating equilibria which satisfy $\underline{K} \ge \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1*}$.

PROOF: By contradiction, suppose there exists a separating equilibrium such that $\underline{K} \ge \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1*}$. Then by (SE3),

$$Q_{1l}^{1*} = \frac{a^1 - c_1}{2b^1} - \frac{1}{2}Q_2^{1*}. \qquad \text{(A1)}$$

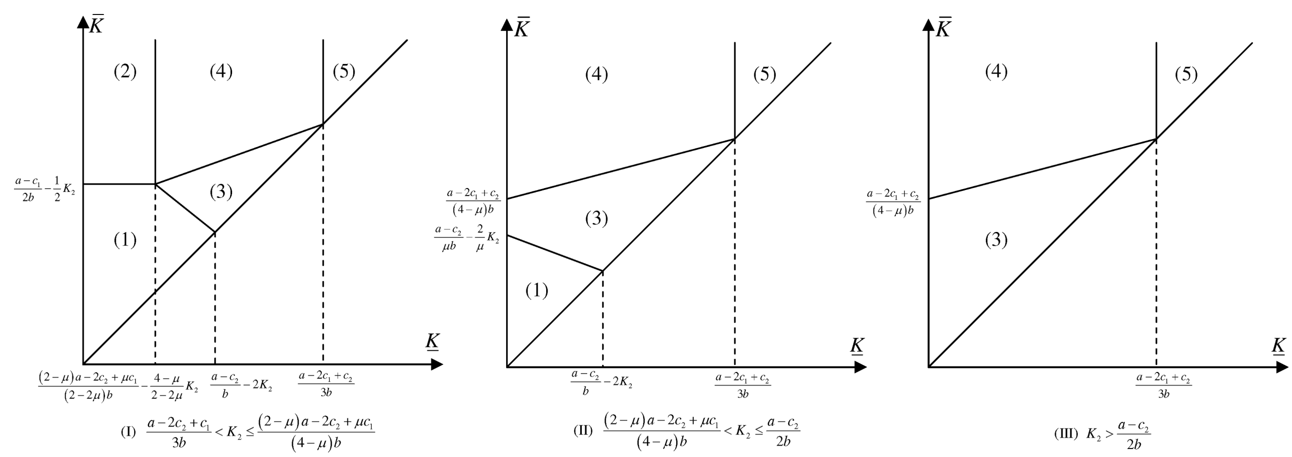

Figure 6. Equilibrium partition when $K_2 > \frac{a-2c_2+c_1}{3b}$.

From (SE4),

$$Q_2^{1^*} \le \frac{a^1 - c_2}{2b^1} - \frac{\mu Q_{1h}^{1^*} + (1-\mu) Q_{1l}^{1^*}}{2}$$
$$= \frac{a^1 - c_2}{2b^1} - \frac{1}{2}\left[\mu Q_{1h}^{1^*} + (1-\mu)\left(\frac{a^1 - c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}\right)\right].$$

Therefore,

$$Q_2^{1^*} \le \frac{(1+\mu)a^1 - 2c_2 + (1-\mu)c_1}{(3+\mu)b^1} - \frac{2\mu}{3+\mu} Q_{1h}^{1^*}. \quad \text{(A2)}$$

Substituting $Q_2^{1^*}$ into (A1),

$$\frac{a^1 - 2c_1 + c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu} Q_{1h}^{1^*} \le Q_{1l}^{1^*}.$$

Because $Q_{1l}^{1^*} \le \underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$, using each of the inequalities:

$$\frac{a^1 - 2c_1 + c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu} Q_{1h}^{1^*} \le \underline{K}$$
$$\Rightarrow Q_{1h}^{1^*} \le \frac{3+\mu}{\mu}\underline{K} - \frac{a^1 - 2c_1 + c_2}{\mu b^1}, \quad \text{(A3)}$$
$$\frac{a^1 - 2c_1 + c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu} Q_{1h}^{1^*} < \frac{a^1 - 2c_1 + c_2}{3b^1}$$
$$\Rightarrow Q_{1h}^{1^*} < \frac{a^1 - 2c_1 + c_2}{3b^1}. \quad \text{(A4)}$$

1. If $Q_{1h}^{1^*} > Q_{1l}^{1^*}$, then $Q_{1h}^{1^*} > \frac{a^1-2c_1+c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu} Q_{1h}^{1^*} \Rightarrow Q_{1h}^{1^*} > \frac{a^1-2c_1+c_2}{3b^1}$, contradicting (A4).
2. Consider $Q_{1h}^{1^*} < Q_{1l}^{1^*}$. If this is the equilibrium, it implies that Firm 1 cannot be better off by producing above $\underline{K}$ (the second-period profits are the same, as the firm is identified as high type, whereas any deviation from equilibrium decision in Period 1 cannot be beneficial). That is, $\pi_1^1(Q_{1h}^{1*}, Q_2^{1^*}) \ge \pi_1^1(\underline{K}, Q_2^{1^*})$. It yields $\frac{a^1-c_1}{b^1} \le \underline{K} + Q_{1h}^{1*} + Q_2^{1^*}$. Combining the above with (A2), we obtain $Q_{1h}^{1*} \ge \frac{2}{3-\mu}\frac{a^1-2c_1+c_2}{b^1} - \frac{3+\mu}{3-\mu}\underline{K}$. Further comparison with (A3) yields $\underline{K} \ge \frac{a^1-2c_1+c_2}{3b^1}$, contradicting the initial assumption $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$.

Therefore, if $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$, there are no separating equilibria such that $\underline{K} \ge \frac{a^1-c_1}{2b^1} - \frac{1}{2} Q_2^{1*}$. □

Proof [of Theorem 1]: Suppose type θ produces $Q_{1\theta}^{1*}$, for $\theta = h, l$, and Firm 2 produces $Q_2^{1^*}$ in the first period under equilibrium. The results are shown separately for two cases: $K_2 \le \frac{a^2-2c_2+c_1}{3b^2}$ and $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$.

Case 1: $K_2 \le \frac{a^2-2c_2+c_1}{3b^2}$. Lemma 1 indicates that in this region, $\pi_{1h}^{2^*}(\mu)$ and $\pi_{1l}^{2^*}(\mu)$ are independent of μ. Therefore, Firm 1 of each type produces its one-period (myopic) best response quantity in the first period, as defined in Eq. (1)–(3):

$$Q_{1h}^{1^*} = Q_{1h}^{1^*}(\mu) = \min\left\{\overline{K}, \frac{a^1 - c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}\right\},$$
$$Q_{1l}^{1^*} = Q_{1l}^{1^*}(\mu) = \min\left\{\underline{K}, \frac{a^1 - c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}\right\},$$
$$Q_2^{1^*} = Q_2^{1^*}(\mu) = \min\left\{K_2, \frac{a^1 - c_2}{2b^1} - \frac{1}{2}\left[\mu Q_{1h}^{1*} + (1-\mu) Q_{1l}^{1^*}\right]\right\}.$$

The two quantities, $Q_{1h}^{1^*}$ and $Q_{1l}^{1^*}$, are clearly equal when $\underline{K} \ge \frac{a^1-c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}$, or after substituting $Q_2^{1^*}$, $\underline{K} \ge \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, which implies a unique pooling equilibrium. Otherwise, $Q_{1l}^{1^*} < Q_{1h}^{1^*}$ and the equilibrium must be separating.

Furthermore, when $\underline{K} \ge \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, the unique pooling equilibrium $Q_{1l}^{1^*} = Q_{1h}^{1^*} = Q_1^{1^*}$ can survive the Intuitive Criterion because any deviation to Q_1^1 yields

$$\Pi_{1l}^*(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu) = \pi_1^1(Q_1^{1*}, Q_2^{1*}) + \pi_{1l}^{2*}(\mu)$$
$$> \pi_1^1(Q_1^1, Q_2^{1*}) + \pi_{1l}^{2*}(1) = \Pi_{1l}(Q_1^1, Q_2^{1^*}, \mu' = 1)$$
$$\Pi_{1h}^*(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu) = \pi_1^1(Q_1^{1*}, Q_2^{1*}) + \pi_{1h}^{2*}(\mu)$$
$$> \pi_1^1(Q_1^1, Q_2^{1*}) + \pi_{1h}^{2*}(1) = \Pi_{1h}(Q_1^1, Q_2^{1^*}, \mu' = 1).$$

where the inequalities hold because $\pi_1^1(Q_1^1, Q_2^{1*})$ is maximized at $Q_1^1 = Q_1^{1*}$ and $\pi_{1\theta}^{2*}(\mu)$ is independent of μ for $\theta = h, l$.

Case 2: $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$. We prove this part in two cases, based on the value of $\underline{K}$.

a. $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$. We show that there are no intuitive pooling equilibria in (a1) below, and then, in part (a2), we show that there exists a unique separating equilibrium.
   a1. Suppose there exists a pooling equilibrium in the first period, where each type produces $Q_1^{1^*}$. We must have $\pi_1^{1^*}(\underline{K}, Q_2^{1^*}) \le \pi_1^{1^*}(Q_1^{1^*}, Q_2^{1^*})$. Otherwise, the high type, by producing a quantity a little bit higher than low type's capacity $\underline{K}$, obtains higher profits in Period 1 and also equal or higher profits in Period 2 (as it separates itself from low type). Therefore,

$$[a^1 - c_1 - b^1(\underline{K} + Q_1^{1^*} + Q_2^{1^*})](\underline{K} - Q_1^{1^*})$$
$$\le 0, \text{ implying } \frac{a^1 - c_1}{b^1} \le \underline{K} + Q_1^{1^*} + Q_2^{1^*} \le \underline{K} + Q_1^{1^*} + \frac{a^1 - c_2}{2b^1} - \frac{1}{2} Q_1^{1^*},$$

   where the last inequality follows from the constraint (PE3). Thus,

$$\frac{a^1 - 2c_1 + c_2}{2b^1} \le \underline{K} + \frac{1}{2} Q_1^{1^*} \le \frac{3}{2}\underline{K}, \text{ implying}$$
$$\underline{K} \ge \frac{a^1 - 2c_1 + c_2}{3b^1},$$

   which is contradictory to the assumption that $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$. Therefore, no pooling equilibria exist when $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$.
   a2. From Lemma A1, if the separating equilibrium exists, it must satisfy $\underline{K} < \frac{a^1-c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}$, thus $\underline{K} < \min\{\overline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}\}$. From (SE3), $Q_{1l}^{1^*} = \min\{\underline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}\} = \underline{K}$ (which is a unique solution). For the high type, the myopic response is optimal in Period 1 and identifies its type (making profits in Period 2 undominated), $Q_{1h}^{1*} = \min\{\overline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2} Q_2^{1^*}\}$. Also, from (SE5), $Q_2^{1^*} = \min\{K_2 \frac{a^1-c_2}{2b^1} - \frac{1}{2}[\mu Q_{1h}^{1*} + (1-\mu) Q_{1l}^{1^*}]\}$. It is easy to show that there is a unique solution, which is separating. That is, each firm produces its myopic quantity in the first period, when $\underline{K} < \frac{a^1-2c_1+c_2}{3b^1}$.

b. If $\underline{K} \ge \frac{a^1-2c_1+c_2}{3b^1}$, then $\underline{K} \ge \frac{a^2-2c_1+c_2}{3b^2}$ because $\frac{a^1-2c_1+c_2}{3b^1} \ge \frac{a^2-2c_1+c_2}{3b^2}$. By Lemma 1, in the second period Firm 2's belief about Firm 1's type does not affect Firm 1's equilibrium profits. Therefore, each player will play its one-period (myopic) best response quantity in the first period. Similarly to the Case 1 that $K_2 \le \frac{a^2-2c_2+c_1}{3b^2}$, the equilibrium is pooling (i.e., $Q_{1h}^{1*} = Q_{1l}^{1*}$) when $\underline{K} \ge \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, and separating (i.e., $Q_{1h}^{1*} > Q_{1l}^{1*}$) when $\underline{K} < \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$. Using the same procedure in the proof for the Case 1, we can show that the unique pooling equilibrium survives the Intuitive Criterion.

In summary, when $\underline{K} < \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, there exists a unique separating equilibrium, and when $\underline{K} \ge \max\{\frac{a^1-2c_1+c_2}{3b^1}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2\}$, there exists a unique intuitive pooling equilibrium.

PROOF OF THEOREM 3. We start by introducing a technical lemma which we will use in the proof of Theorem 3. □

LEMMA A2: If $\frac{a^1-2c_1+c_2}{3b^1} < \frac{a^2-2c_1+c_2}{3b^2}$ and $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$ and $\underline{K} \ge \frac{a^1-2c_1+c_2}{3b^1}$, then there can be no separating equilibria which satisfy $\underline{K} < \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}$.

PROOF: By contradiction, suppose that there exists a separating equilibrium such that $\underline{K} < \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}$. Then by (SE3), $Q_{1l}^{1^*} = \min\{\underline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}\} = \underline{K}$.

Because $\underline{K} < \min\{\overline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}\}$, $Q_{1h}^{1*} = \min\{\overline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}\}$.

By (SE4), $Q_2^{1^*} = \min\{K_2, \frac{a^1-c_2}{2b^1} - \frac{\mu Q_{1h}^{1*}+(1-\mu)Q_{1l}^{1^*}}{2}\}$.

It is easy to verify that there are no solutions for the above three equations. Therefore, there can be no separating equilibria which satisfy $\underline{K} < \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}$. □

PROOF OF THEOREM 3:

a. The proof is similar to the proof of Theorem 1(a). Given that $\underline{K}$ is very small, high-type Firm 1, even without considering its desire to distinguish itself from low type, already produces quantity that exceeds $\underline{K}$, the maximum low type can produce.
b. If $\underline{K} \ge \frac{a^2-2c_1+c_2}{3b^2}$ or $\{\overline{K} \le \frac{a^2-c_2}{b^2} - 2K_2$ and $\underline{K} \ge \frac{a^1-2c_1+c_2}{3b^1})\}$, then $\pi_{1h}^{2^*}(\mu)$ and $\pi_{1l}^{2^*}(\mu)$ are independent of μ by Lemma 1 and each firm produces its one-period (i.e., myopic) best response quantity in the first period. In this case, $\underline{K} \ge \frac{a^1-2c_1+c_2}{3b^1}$. Therefore, each firm produces its "uncapacitated" equilibrium quantity, that is, $Q_{1h}^{*} = Q_{1l}^{1^*} = \frac{a^1-2c_1+c_2}{3b^1}$ and $Q_2^{1^*} = \frac{a^1-2c_1+c_2}{3b^1}$. It is easy to show that in both subcases this unique pooling equilibrium survives the Intuitive Criterion following the same procedure for the Case 1 in the proof of Theorem 1.
c. We prove this part in two steps. First in (c1) we show that there are no intuitive pooling equilibria, then in (c2) we show that there is a unique separating equilibrium.
   c1. We consider the following two possibilities:
1. If $\Pi_{1h}(\underline{K}, Q_2^{1^*}, \mu' = 1) > \Pi_{1h}(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu)$, the high type can obtain higher profit by producing slightly above low-type's capacity $\underline{K}$. As the high type would surely deviate from $Q_1^{1^*}$, no pooling equilibria are possible.
2. If $\Pi_{1h}(\underline{K}, Q_2^{1^*}, \mu' = 1) \le \Pi_{1h}(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu)$, then rearranging the terms

$$\pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(\mu) \le \pi_1(Q_1^{1^*}, Q_2^{1^*}) - \pi_1(\underline{K}, Q_2^{1^*}). \quad (A5)$$

By Lemma 2, for any $\mu \in [0, 1)$,

$$\pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(\mu) > \pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(\mu). \quad (A6)$$

Inequality (A6) implies that the extra profit of high type, by successfully differentiating its type, is strictly greater than the extra profit of low type by pretending to be high type (which acts as a motive for high type to seek differentiating actions). Suppose there exists a pooling equilibrium with period-one production $Q_1^{1^*}$. Denote by ΔC the decrease in profit of Firm 1 in the first period, when it chooses $Q_1^1 \ne Q_1^{1^*}$, $\Delta C = \pi_1^1(Q_1^{1^*}, Q_2^{1^*}) - \pi_1^1(Q_1^1, Q_2^{1^*})$.

By (A5) and (A6), there exist $Q_1^1 \in [0, \underline{K}]$ such that

$$\pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(\mu) < \Delta C < \pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(\mu).$$

We immediately have

$$\begin{aligned}
\Pi_{1l}(Q_1^1, Q_2^{1^*}, \mu' = 1) &= \pi_1^1(Q_1^1, Q_2^{1^*}) + \pi_{1l}^{2^*}(1) \\
&< \pi_1^1(Q_1^{1^*}, Q_2^{1^*}) + \pi_{1l}^{2^*}(\mu) \\
&= \Pi_{1l}^*(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu), \\
\Pi_{1h}(Q_1^1, Q_2^{1^*}, \mu' = 1) &= \pi_1^1(Q_1^1, Q_2^{1^*}) + \pi_{1h}^{2^*}(1) \\
&> \pi_1^1(Q_1^{1^*}, Q_2^{1^*}) + \pi_{1h}^{2^*}(\mu) \\
&= \Pi_{1h}^*(Q_1^{1^*}, Q_2^{1^*}, \mu' = \mu),
\end{aligned}$$

which fails the Intuitive Criterion. Therefore, no intuitive pooling equilibria are possible.

c2. By Lemma A2, when $K_2 > \frac{a^2-2c_2+c_1}{3b^2}$ and $\frac{a^1-2c_1+c_2}{3b^1} \le \underline{K} < \frac{a^2-2c_1+c_2}{3b^2}$ and $\overline{K} > \frac{a^2-c_2}{b^2} - 2K_2$, if the separating equilibrium exists, it must satisfy $\underline{K} \ge \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}$.

By (SE3) and (SE4),

$$Q_{1l}^{1^*} = \min\left\{\underline{K}, \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}\right\} = \frac{a^1-c_1}{2b^1} - \frac{1}{2}Q_2^{1^*}, \quad (A7)$$

$$Q_2^{1^*} = \min\left\{K_2, \frac{a^1-c_2}{2b^1} - \frac{1}{2}[\mu Q_{1h}^{1*} + (1-\mu)Q_{1l}^{1^*}]\right\}. \quad (A8)$$

If the separating equilibrium exists, by (SE1), low type incurs higher cost in period one, when mimicking high type, than the potential profit from mimicking high type in Period 2:

$$\pi_1^1(Q_{1l}^{1^*}, Q_2^{1^*}) - \pi_1^1(Q_{1h}^{1*}, Q_2^{1^*}) \ge \pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0).$$

Because low type produces myopic quantity, at a separating equilibrium the high type must deviate from the myopic quantity. By (SE2), the cost of the deviation in the first period is less than the cost of being mistaken for low type in the second period:

$$\pi_1^1(Q_{1l}^{1^*}, Q_2^{1^*}) - \pi_1^1(Q_{1h}^{1*}, Q_2^{1^*}) \le \pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(0).$$

Because $\pi_1^1(Q_{1l}^{1^*}, Q_2^{1^*}) - \pi_1^1(Q_{1h}^{1*}, Q_2^{1^*}) = b^1(Q_{1h}^{1*} - Q_{1l}^{1^*})^2$,

$$\pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0) \le b^1(Q_{1h}^{1*} - Q_{1l}^{1^*})^2 \le \pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(0). \quad (A9)$$

From Lemma 2, $\pi_{1h}^{2^*}(1) - \pi_{1h}^{2^*}(0) > \pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0)$, when $\frac{a^2-2c_2+c_1}{3b^2} < K_2 \le \frac{a^2-c_2}{2b^2}$, $\frac{a^1-2c_1+c_2}{3b^1} \le \underline{K} < \frac{a^2-2c_1+c_2}{3b^2}$ and $\overline{K} > \frac{a^2-c_2}{b^2} - 2K_2$. Therefore, there exists $(Q_{1h}^{1^*}, Q_{1l}^{1^*})$ which satisfies (A9). Clearly, $Q_{1h}^{1^*} \ne Q_{1l}^{1^*}$, as $\pi_{1l}^{2^*}(1) - \pi_{1l}^{2^*}(0) > 0$.

For a separating equilibrium to survive the Intuitive Criterion, the separating quantities need satisfy the following equation:

$$b^1(Q_{1h}^{1*} - Q_{1l}^{1*})^2 = \pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0). \tag{A10}$$

(Otherwise, suppose that there exists a separating equilibrium $(Q_{1h}^{1*}, Q_{1l}^{1*})$ such that $b^1(Q_{1h}^{1*} - Q_{1l}^{1*})^2 > \pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)$. By the continuity of the profit functions, we can find a quantity Q_1^1 which is between Q_{1h}^{1*} and Q_{1l}^{1*} and satisfies $\pi_1^1(Q_{1l}^{1*}, Q_2^{1*}) - \pi_1^1(Q_1^1, Q_2^{1*}) > \pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)$. The low-type Firm 1 will not deviate to this quantity Q_1^1 because

$$\pi_1^1(Q_{1l}^{1*}, Q_2^{1*}) + \pi_{1l}^{2*}(0) > \pi_1^1(Q_1^1, Q_2^{1*}) + \pi_{1l}^{2*}(1)$$

where the left hand side is the low-type's equilibrium profit over two periods and the right-hand side is the maximum profit over two periods for the low-type firm by deviating to Q_1^1. The high type is able to signal its type by producing Q_1^1, and then increase its total profits over two periods because

$$\pi_1^1(Q_1^1, Q_2^{1*}) + \pi_{1h}^{2*}(1) > \pi_1^1(Q_{1h}^{1*}, Q_2^{1*}) + \pi_{1h}^{2*}(1)$$

where the inequality holds because Q_1^1 is closer to the myopic optimum Q_{1l}^{1*} than Q_{1h}^{1*}. Therefore, any separating equilibria $(Q_{1h}^{1*}, Q_{1l}^{1*})$ such that $b^1(Q_{1h}^{1*} - Q_{1l}^{1*})^2 > \pi_{1l}^{2*}(1) - \pi_{1l}^{2*}(0)$ fail the Intuitive Criterion.)

Then, $Q_{1h}^{1*} = \overline{Q}_{1h} = \min\{\underline{K}, Q_{1l}^{1*} + \sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}\}$ (because $\mu'(Q_1^1) = 1$ if $Q_1^1 > \underline{K}$) or $Q_{1h}^{1*} = \underline{Q}_{1h} = Q_{1l}^{1*} - \sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}$.

1. If $Q_{1l}^{1*} < \sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{2l}^{*}(0)}{b^1}}$, then $\underline{Q}_{1h}$ would have to be negative, which is not implementable. Therefore $Q_{1h}^{1*} = \overline{Q}_{1h}$.
2. If $Q_{1l}^{1*} \geq \sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}$, then $Q_{1h}^{1*} = \overline{Q}_{1h} = \min\{\underline{K}^+, Q_{1l}^{1*} + \sqrt{\frac{\pi_{2l}^{*}(1)-\pi_{1l}^{2*}(0)}{b^1}}\}$ or $Q_{1h}^{1*} = \underline{Q}_{1h} = Q_{1l}^{1*} - \sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}$. Note that high type is usually not indifferent between underproducing by producing $\underline{Q}_{1h}$ and overproducing by producing $\overline{Q}_{1h}$, and high type actually loses the same or less by overproducing:

$$\begin{aligned}&\pi_1^1(\overline{Q}_{1h}, Q_2^{1*}) - \pi_1^1(\underline{Q}_{1h}, Q_2^{1*})\\&= [a^1 - c_1 - b^1(\overline{Q}_{1h} + \underline{Q}_{1h} + Q_2^{1*})](\overline{Q}_{1h} - \underline{Q}_{1h})\\&\geq [a^1 - c_1 - b^1(2Q_{1l}^{1*} + Q_2^{1*})](\overline{Q}_{1h} - \underline{Q}_{1h}) = 0\end{aligned}$$

Therefore $Q_{1h}^{1*} = \overline{Q}_{1h}$.

Solving Eq. (A7) and (A8) yields

$$Q_{1l}^{1*} = \begin{cases} \frac{a^1-c_1}{2b^1} - \frac{1}{2}K_2 & \text{if } Q_{1h}^{1*} < \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{2\mu b^1} - \frac{3+\mu}{2\mu}K_2 \\ \frac{a^1-2c_1+c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu}Q_{1h}^{1*} & \text{if } Q_{1h}^{1*} \geq \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{2\mu b^1} - \frac{3+\mu}{2\mu}K_2 \end{cases},$$

$$Q_2^{1*} = \begin{cases} K_2 & \text{if } Q_{1h}^{1*} < \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{2\mu b^1} - \frac{3+\mu}{2\mu}K_2 \\ \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{(3+\mu)b^1} - \frac{2\mu}{3+\mu}Q_{1h}^{1*} & \text{if } Q_{1h}^{1*} \geq \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{2\mu b^1} - \frac{3+\mu}{2\mu}K_2 \end{cases}.$$

It is straightforward to show that the solution where $Q_{1h}^{1*} < \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{2\mu b^1} - \frac{3+\mu}{2\mu}K_2$ is not feasible, and therefore the final optimal quantities are given by $Q_{1l}^{1*} = \frac{a^1-2c_1+c_2}{(3+\mu)b^1} + \frac{\mu}{3+\mu}Q_{1h}^{1*}$ and $Q_2^{1*} = \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{(3+\mu)b^1} - \frac{2\mu}{3+\mu}Q_{1h}^{1*}$. Therefore, when $\frac{a^2-2c_2+c_1}{3b^2} < K_2 \leq \frac{a^2-c_2}{2b^2}$, $\frac{a^1-2c_1+c_2}{3b^1} \leq \underline{K} < \frac{a^2-2c_1+c_2}{3b^2}$ and $\overline{K} > \frac{a^2-c_2}{b^2} - 2K_2$, at the separating equilibrium,

$$\begin{aligned} Q_{1h}^{1*} &= \min\left\{\underline{K}^+, \frac{a^1-2c_1+c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}\right\}, \\ Q_{1l}^{1*} &= \frac{a^1-2c_1+c_2}{(3+\mu)b^1} \\ &\quad + \frac{\mu}{3+\mu}\min\left\{\underline{K}^+, \frac{a^1-2c_1+c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}\right\}, \\ Q_2^{1*} &= \frac{(1+\mu)a^1-2c_2+(1-\mu)c_1}{(3+\mu)b^1} \\ &\quad - \frac{2\mu}{3+\mu}\min\left\{\underline{K}^+, \frac{a^1-2c_1+c_2}{3b^1} + \frac{3+\mu}{3}\sqrt{\frac{\pi_{1l}^{2*}(1)-\pi_{1l}^{2*}(0)}{b^1}}\right\}. \end{aligned}$$

□

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